MANDATORY APPENDIX XXII

ARTICLE XXII-1000

RULES FOR REINFORCEMENT OF CONE-TO-CYLINDER JUNCTION UNDER EXTERNAL PRESSURE

XXII-1100 INTRODUCTION

XXII-1110 SCOPE

(a) The equations of this Appendix provide for the design of reinforcement, if needed, at the cone-to-cylinder junctions for reducer sections and conical heads where all the elements have a common axis and the half-apex angle \( \alpha \leq 60 \) deg. Subparagraph XXII-1300(d) provides for special analysis in the design of cone-to-cylinder intersections with or without reinforcing rings where \( \alpha \) is greater than 60 deg.

(b) In the design of reinforcement for a cone-to-cylinder juncture, the requirements of ND-3336 shall be met.

XXII-1200 NOMENCLATURE

The nomenclature given below is used in the equations of the following subparagraphs:

- \( A = \) factor determined from the applicable chart in Section II, Part D, Subpart 3 for the material used in the stiffening ring, corresponding to the factor \( B \), below, and the design temperature for the shell under consideration
- \( A_e = \) effective area of reinforcement due to excess metal thickness
- \( A_{RL} = \) required area of reinforcement at large end
- \( A_{RS} = \) required area of reinforcement at small end
- \( A_T = \) cross-sectional area of the stiffening ring
- \( A_{TE} = \) equivalent area of cylinder, cone, and stiffening ring
  
  \[
  A_{TE} = \frac{D_L}{2} \left( \frac{b_T}{2} + \frac{b_C}{2} \right) + A_p, \text{ for large end}
  \]
  
  \[
  A_{TS} = \frac{D_S}{2} \left( \frac{b_T}{2} + \frac{b_S}{2} \right) + A_p, \text{ for small end}
  \]
- \( B = \) factor determined from the applicable chart in Section II, Part D, Subpart 3 for the material used for the stiffening
- \( B = \) factor determined from the applicable chart in Section II, Part D, Subpart 3 for the material used for the stiffening
- \( D_L = \) outside diameter of large end of conical section under consideration
- \( D_S = \) outside diameter of cylindrical shell (In conical shell calculations, the value of \( D_s \) and \( D_L \) should be used in calculations in place of \( D_s \) depending on whether the small end \( D_s \), or large end \( D_L \), is being examined.)
- \( D_L = \) outside diameter at small end of conical section under consideration
- \( E = \) lowest efficiency of the longitudinal joint in the shell or head or of the joint in the reducer; \( E = 1 \) for butt welds in compression
- \( E_c = \) modulus of elasticity of cone material
- \( E_R = \) modulus of elasticity of reinforcing material
- \( E_s = \) modulus of elasticity of shell material
- \( E_x = E_c, E_R, \text{ or } E_s \)
- \( f_1 = \) axial load at large end (excluding pressure \( P \)), lb/in. (N/mm)
- \( f_2 = \) axial load at small end (excluding pressure \( P \)), lb/in. (N/mm)
- \( I_s = \) required moment of inertia of the stiffening ring cross section about its neutral axis parallel to the axis of the shell
- \( I_{s'} = \) required moment of inertia of the combined ring-shell-cone cross section about its neutral axis parallel to the axis of the shell, in.\(^4\) (mm\(^4\)). The width of shell which is taken as contributing to the moment of inertia of the combined section shall not be greater than 1.10 \( \sqrt{D_o T} \) and shall be taken as lying one-half on each side of the centroid of the ring. Portions of the shell plate shall not be considered as contributing area to more than one stiffening ring. If the stiffeners should be so located that the maximum permissible effective shell sections overlap on either or both sides of a stiffener, the effective shell section for that stiffener shall be shortened by one-half of each overlap.
- \( L = \) axial length of cone

\[ L = \frac{S_E}{5P_{ER}} \text{ but not less than } 1.5 \]
\[ L_c = \text{length of cone between stiffening rings measured along surface of cone. For cones without intermediate stiffeners,} \]
\[ L_c = \sqrt{l^2 + (R_L - R_s)^2} \]

\[ L_L = \text{design length of a vessel section taken as the largest of the following:} \]
\[ (a) \text{the center-to-center distance between the cone-to-large-shell junction and an adjacent stiffening ring on the large shell;} \]
\[ (b) \text{the distance between the cone-to-large-shell junction and one-third the depth of head on the other end of the large shell if no other stiffening rings are used.} \]

\[ L_s = \text{design length of a vessel section taken as the largest of the following:} \]
\[ (a) \text{the center-to-center distance between the cone-to-small-shell junction and an adjacent stiffening ring on the small shell;} \]
\[ (b) \text{the distance between the cone-to-small-shell junction and one-third the depth of the head on the other end of the small shell if no other stiffening rings are used.} \]

\[ P = \text{external design pressure} \]
\[ Q_L = \frac{P R_L}{2} + f_1; \text{axial compressive force at large end due to pressure and } f_1, \text{ lb/in. (N/mm)} \]
\[ Q_s = \frac{P R_s}{2} + f_2; \text{axial compressive force at small end due to pressure and } f_2, \text{ lb/in. (N/mm)} \]
\[ R_L = \text{inside radius of large cylinder} \]
\[ R_s = \text{inside radius of small cylinder} \]
\[ S' = \text{the lesser of twice the allowable stress at design metal temperature from Section II, Part D, Subpart 1, Tables 1A and 1B or 0.9 times the tabulated yield strength at design metal temperature from Section II, Part D, Subpart 2, Tables Y-1 and Y-2} \]
\[ S_R = \text{allowable stress of reinforcing material} \]
\[ S_s = \text{allowable stress of shell} \]
\[ T = \text{minimum required thickness of cylinder at cone-to-cylinder junction, exclusive of corrosion allowance} \] (see ND-3133.3)
\[ T_c = \text{nominal thickness of cone at cone-to-cylinder junction, exclusive of corrosion allowance} \] (see ND-3121)
\[ T_l = \text{the smaller of } (T_s - T) \text{ or } (T_c - T_s) \]
\[ T_r = \text{minimum required thickness of cone at cone-to-cylinder junction, exclusive of corrosion allowance} \] (see ND-3121)
\[ T_s = \text{nominal thickness of cylinder at cone-to-cylinder junction, exclusive of corrosion allowance} \] (see ND-3121)
\[ \alpha = \text{one-half the included (apex) angle of the cone at the centerline of the head} \]
\[ \Delta = \text{value to indicate need for reinforcement at cone-to-cylinder intersection having a half-apex angle } \alpha \leq 60 \text{ deg. When } \Delta \geq \alpha, \text{ no reinforcement is required at the junction (see Table XXII-1200-1).} \]

**XXII-1300 DESIGN PRESSURE**

(a) Reinforcement shall be provided at the junction of the cone with the large cylinder for conical heads and reducers without knuckles when the value of \( \Delta \) obtained from Table XXII-1200-1 using the appropriate ratio \( P/S'E \), is less than \( \alpha \). Interpolation may be made in the Table.

The cross-sectional area of the reinforcement ring shall be at least equal to that indicated by the following equation:
\[ A_{rL} = \frac{k Q_L R_l \tan \alpha}{S'E} \left[ 1 - \frac{1}{4} \left( \frac{P R_L - Q_L}{Q_L} \right) \Delta^2 \right] \tag{1} \]

When the thickness, less corrosion allowance, of both the reducer and cylinder exceeds that required by the applicable design equations, the minimum excess thickness may be considered to contribute to the required reinforcement ring in accordance with the following equation:
\[ A_e = 4 T_b \sqrt{R_l T_s} \]  

Any additional area of reinforcement which is required shall be situated within a distance of \( \sqrt{R_l T_s} \) from the junction of the reducer and the cylinder. The centroid of the added area shall be within a distance of 0.5 \( \sqrt{R_l T_s} \) from the junction.

The reinforcement ring at the cone-to-cylinder junction shall also be considered as a stiffening ring. The required moment of inertia of a circumferential stiffening ring cross section shall not be less than that determined by the following equation:
\[ I_s = \frac{A D_L^2 A_{rL}}{14.0} \]

<table>
<thead>
<tr>
<th>Table XXII-1200-1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Values of ( \Delta ) for Junctions at the Large Cylinder for ( \alpha \leq 60 \text{ deg} )</strong></td>
</tr>
<tr>
<td>( P/S'E )</td>
</tr>
<tr>
<td>( \Delta ), deg</td>
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<td>( P/S'E )</td>
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<td>( \Delta ), deg</td>
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<td>( P/S'E )</td>
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<td>( \Delta ), deg</td>
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</tbody>
</table>

**NOTE:**
(1) \( \Delta = 60 \text{ deg for greater values of } P/S'E \).
The required moment of inertia of the combined ring-shell-core cross section shall not be less than that determined by the following equation:

$$I'_s = \frac{AD_T^2AT_L}{10.9}$$

The moment of inertia for a stiffening ring at the large end shall be determined by the following procedure:

**Step 1.** Assuming that the shell has been designed and $D_L$, $L_L$, and $T$ are known, select a member to be used for the stiffening ring and determine cross-sectional area $A_{TL}$. Then calculate factor $B$ using the following equation:

$$B = \frac{3}{4}\left( \frac{F_LD_k}{A_{TL}} \right)$$

where

$$F_L = PM + f_1 \tan \alpha$$

$$M = \frac{R_P \tan \alpha}{2} + \frac{L_D}{2} + \frac{R_f^2 - R_s^2}{3R_L \tan \alpha}$$

$A_{TL}$ was defined previously.

**Step 2.** Enter the right-hand side of the applicable material chart in Section II, Part D, Subpart 3 for the material under consideration at the value of $B$ determined by Step 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of $A$ in Step 4, below.

**Step 3.** Move horizontally to the left to the material/temperature line for the design metal temperature. For values of $B$ falling below the left end of the material/temperature line, see Step 5.

**Step 4.** Move vertically to the bottom of the chart and read the value of $A$.

**Step 5.** For value of $B$ falling below the left end of the material/temperature line for the design temperature, the value of $A$ can be calculated using the formula $A = 2B/E_s$.

**Step 6.** Compute the value of the required moment of inertia from the equations for $I_s$ or $I_s'$ above.

**Step 7.** Calculate the available moment of inertia of the stiffening ring using the section corresponding to that used in Step 6.

**Step 8.** If the required moment of inertia is greater than the moment of inertia for the section selected in Step 1, a new section with a larger moment of inertia must be selected and a new moment of inertia determined. If the required moment of inertia is smaller than the moment of inertia of the section selected in Step 1, that section is satisfactory.

The requirements of ND-4430 are to be met in attaching stiffening rings to the shell.

(b) Reinforcement shall be provided at the junction of the conical shell of a reducer without a flare and the small cylinder. The cross-sectional area of the reinforcement ring shall be at least equal to that indicated by the following formula:

$$A_{RS} = \frac{kQ_sR_s \tan \alpha}{S'E}$$  \hspace{1cm} (3)

When the thickness, less corrosion allowance, of either the reducer or cylinder exceeds that required by the applicable design formula, the thickness may be considered to contribute to the required reinforcement ring in accordance with the following formula:

$$A_e = \sqrt{R_sT_e} \left( T_e - T \right) + \sqrt{R_sT_s} \left( T_s - T \right)$$  \hspace{1cm} (4)

Any additional area of reinforcement which is required shall be situated within a distance of $\sqrt{R_sT_s}$ from the junction, and the centroid of the added area shall be within a distance of $0.5\sqrt{R_sT_s}$ from the junction.

The reinforcement ring at the cone-to-cylinder junction shall also be considered as a stiffening ring. The required moment of inertia of a circumferential stiffening ring cross section shall not be less than that determined by the following equation:

$$I_s = \frac{AD_s^2AT_s}{14.0}$$

The required moment of inertia of the combined ring-shell-cone cross section shall not be less than that determined by the following equation:

$$I'_s = \frac{AD_s^2AT_s}{10.9}$$

The moment of inertia for a stiffening ring at the small end shall be determined by the following procedure:

**Step 1.** Assuming that the shell has been designed and $D_s$, $L_s$, and $T$ are known, select a member to be used for the stiffening ring and determine cross-sectional area $A_{Ts}$. Then calculate factor $B$ using the following equation:

$$B = \frac{3}{4}\left( \frac{F_sD_k}{A_{Ts}} \right)$$

where

$$F_s = PN + f_2 \tan \alpha$$

$$N = \frac{R_s \tan \alpha}{2} + \frac{L_s}{2} + \frac{R_f^2 - R_s^2}{3R_L \tan \alpha}$$

$A_{Ts}$ was defined previously.

**Step 2.** Enter the right-hand side of the applicable material chart in Section II, Part D, Subpart 3 for the material under consideration at the value of $B$ determined by Step 1.
1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of \( A \) in Step 4, below.

*Step 3.* Move horizontally to the left to the material/temperature line for the design metal temperature. For values of \( B \) falling below the left end of the material/temperature line, see Step 5.

*Step 4.* Move vertically to the bottom of the chart and read the value of \( A \).

*Step 5.* For values of \( B \) falling below the left end of the material/temperature line for the design temperature, the value of \( A \) can be calculated using the formula

\[
A = \frac{2B}{E_s}
\]

*Step 6.* Compute the value of the required moment of inertia from the equations for \( I_s \) or \( I_s' \) above.

*Step 7.* Calculate the available moment of inertia of the stiffening ring using the section corresponding to that used in Step 6.

*Step 8.* If the required moment of inertia is greater than the moment of inertia for the section selected in Step 1, a new section with a larger moment of inertia must be selected and a new moment of inertia determined. If the required moment of inertia is smaller than the moment of inertia for the section selected in Step 1, that section is satisfactory.

The requirements of ND-4430 are to be met in attaching stiffening rings to the shell.

(c) Reducers, such as those made up of two or more conical frustums having different slopes, may be designed in accordance with (d) below.

(d) As an alternative to the rules provided in the preceding (a) and (b) and when half the apex angle is greater than 60 deg, the design may be based on special analysis such as numerical methods or the beam-on-elastic-foundation analysis of Timoshenko, Hetenyi, or Watts and Lang. The stresses at the junction shall meet all of the allowable stress limits of this Division. The effect of shell and cone buckling on the required area and moment of inertia at the joint shall also be considered in the analysis. The theoretical buckling pressure of the junction shall be at least 3.3 times the allowable external design pressure of the junction.