ARTICLE A-8000
STRESSES IN PERFORATED FLAT PLATES

A-8100 INTRODUCTION
A-8110 SCOPE

(a) This Article contains a method of analysis for flat perforated plates when subjected to directly applied loads or loadings resulting from structural interaction with adjacent members. This method applies to perforated plates which satisfy the conditions of (1) through (5).

1. The holes are in an array of equilateral triangles.
2. The holes are circular.
3. There are 19 or more holes.
4. The ligament efficiency is greater than 5% ($\eta \geq 0.05$).
5. The plate is thicker than twice the hole pitch ($t/P \geq 2$). If only in-plane loads or thermal skin stresses are considered, this limitation does not apply.

(b) Credit may be taken for the stiffening effect of the tubes in the perforations. The extent to which the tubes stiffen the perforated plate depends on the materials, the tubes in the perforations. The extent to which the tubes are considered, this limitation does not apply.

(c) Credit may be taken for the staying action of the tubes where applicable.

A-8120 NOMENCLATURE

- $c =$ radius of ring load (Figure A-8132.2-1)
- $E =$ Young’s modulus for plate material
- $E^* =$ effective Young’s modulus for perforated plate (Figure A-8131-1)
- $E_t =$ Young’s modulus for tube material
- $h =$ nominal width of ligament at the minimum cross section
- $K =$ stress multiplier for stresses averaged across the width of the ligament but not through the thickness (Figure A-8142-1)
- $K_{m} =$ ratio of peak stress in reduced ligament to the peak stress in normal ligament
- $K_r =$ stress multiplier for circumferential stress in the plate rim (Figure A-8142-6)
- $K_{skin} =$ stress multiplier for thermal skin stress (Figure A-8153-1)
- $\ln =$ logarithm
- $M =$ radial moment acting at edge of plate, in.-lb/in. ($N \cdot mm/mm$) of circumference
- $P =$ nominal distance between hole center lines, pitch
- $P_1, P_2 =$ pressures acting on surfaces of the plate
- $p_r =$ pressure inside tubes
- $p_s =$ pressure on surface where stress is computed, $p_1$ or $p_2$
- $Q =$ radial force acting at edge of plate, lb/in. (N/mm) of circumference
- $r =$ designation of radial location in plate
- $R^* =$ the effective radius of the perforated plate
- $r_o =$ radial distance from center of plate to center of outermost hole
- $S =$ stress intensity (A-8142)
- $t =$ thickness of plate exclusive of cladding or corrosion allowance
- $T_r =$ temperature of the surface of the plate
- $t_r =$ tube wall thickness
- $W =$ total ring load acting on plate (Figure A-8132.2-1), lb (N)
- $w =$ radial displacement of plate edge
- $x =$ axis of symmetry of hole pattern through the smaller ligament thickness (Figures A-8142-3 through A-8142-5)
- $y =$ axis of symmetry of hole pattern, perpendicular to $x$ axis
- $Y =$ stress multiplier for peak ligament stresses (Figure A-8142-1)
- $\Delta p =$ differential pressure across the plate
- $\alpha =$ coefficient of thermal expansion, in./in.-°F (mm/mm-°C)
- $\beta =$ biaxiality ratio ($\sigma_x/\sigma_y$ or $\sigma_y/\sigma_x$ or $\sigma_1/\sigma_2$ or $\sigma_2/\sigma_1$), where $-1 \leq \beta \leq 1$
- $\eta =$ ligament efficiency
- $\theta =$ rotation of plate edge, rad
- $\nu =$ Poisson’s ratio
- $\nu^* =$ effective Poisson’s ratio for perforated plate (Figure A-8131-1)
- $\rho =$ radius of holes in the plate
- $\sigma_{1, 2} =$ principal stress in the plane of the equivalent solid plate (A-8142.2)
- $\sigma_{ovl} =$ larger absolute value of $\sigma_1$ or $\sigma_2$ [A-8142.1(b)]
- $\sigma_r =$ radial stresses in the equivalent solid plate

Delete endnote 18, which says "Express metric values in exponential form"
σ_{rim} = nominal circumferential stress in solid rim  
σ_{skin} = thermal skin stress  
σ_θ = tangential stress in the equivalent solid plate  
σ_r = radial stress averaged through the depth of the equivalent solid plate  

A-8130 ANALYSIS OF CIRCULAR PERFORATED AREA  

A-8131 Procedure  

(a) The analysis method for perforated plates presented in this Article utilizes the concept of the equivalent solid plate. In this method, the perforated plate is replaced by a solid plate which is geometrically similar to the perforated plate but has modified values of the elastic constants.  

(b) The elastic modulus E and Poisson’s ratio ν are replaced by the effective elastic modulus E* and effective Poisson’s ratio ν* of the perforated plate, and conventional equations for plates are used to determine the deformations and nominal stresses for the equivalent solid plate. The deformations so computed may be used directly in evaluating interaction effects. The actual values of the stress intensities in the perforated plate are determined by applying multiplying factors to the nominal stresses computed for the equivalent solid plate.  

(c) The effective elastic constants are functions of the ligament efficiency η. The values are given in Figure A-8131-1 for the range of 0.05 ≤ η ≤ 1.0 in the form of ν* vs. η for a material with ν = 0.3, and E*/E vs. η. The stress multipliers are given in Figures A-8142-1 through A-8142-6. The Y factors presented in Figures A-8142-3 and A-8142-4 represent the largest values occurring through the thickness at the given angular position.  

(d) The region of the perforated plate outside the effective radius R* is called the plate rim. This unperforated portion of the plate may be considered as a separate connecting member, a ring or cylinder, and the structure may be analyzed in accordance with the procedures of A-6000.  

A-8132 Analysis of Equivalent Solid Plate  

In the following subparagraphs, equations are given for the nominal stresses and edge displacements for the equivalent solid circular plate under various axisymmetric load conditions.  

A-8132.1 Edge Loads (see Figure A-8132.1-1).  

(a) Stresses at any location on the surface of the equivalent solid plate.  

\[ \sigma_r = \sigma_\theta = \frac{Q}{t} \pm \frac{6M}{t^2} \]  

(1)  

When double signs are used, the upper sign applies to the top surface as shown in Figure A-8132.1-1.  

(b) Edge displacements of midplane at R*:  

\[ w = \frac{R^*}{E^*}\left(1 - \nu^*\right)\delta \]  

\[ \theta = -\frac{12R^*}{E^*}\left(1 - \nu^*\right)M \]  

(2)  

(3)  

A-8132.2 Ring Loads Transverse to the Plane of the Plate (See Figure A-8132.2-1).  

(a) Stresses at any radial location r on the surfaces of the equivalent solid plate: for \( r \leq c \),  

\[ \sigma_r = \sigma_\theta = \pm \frac{3W}{2\pi t^2} \left[ \frac{1}{2} \left(1 - \nu^*\right) - \frac{1}{2} \left(1 - \nu^*\right) \times \left(\frac{c}{R^*}\right)^2 \right] \]  

\[ + \left(1 + \nu^*\right) \ln\frac{R^*}{c} \]  

(4)  

for \( r > c \),  

\[ \sigma_r = \pm \frac{3W}{2\pi t^2} \left[ \frac{1}{2} \left(1 - \nu^*\right) \left(\frac{c}{r}\right)^2 - \frac{1}{2} \left(1 - \nu^*\right) \times \left(\frac{c}{R^*}\right)^2 \times \left(1 + \nu^*\right) \ln\frac{R^*}{r} \right] \]  

(5)  

\[ \sigma_\theta = \pm \frac{3W}{2\pi t^2} \left[ \left(1 - \nu^*\right) - \frac{1}{2} \left(1 - \nu^*\right) \left(\frac{c}{r}\right)^2 \right] \]  

\[ - \frac{1}{2} \left(1 - \nu^*\right) \times \left(\frac{c}{R^*}\right)^2 \times \left(1 + \nu^*\right) \ln\frac{R^*}{r} \]  

(6)
Figure A-8131-1

Effective Elastic Constants

Ligament Efficiency, $\eta = h/P$

$E^*/E$

$t$ = thickness of perforated plate

$t / P \geq 2$
(b) Edge displacements of midplane at $r = R^*$:

$$w = 0$$  \hspace{1cm} (7)$$

$$g = \frac{3R^*}{\pi E^* t^3} \left[ 1 - \left( \frac{c}{R^*} \right)^2 \right] [W] \hspace{1cm} (8)$$

**A-8132.3 Uniformly Distributed Pressure Loads (See Figure A-8132.3-1).**

(a) Stresses at any location $r$ on the surfaces of the equivalent solid plate:

$$\sigma_r = \pm \frac{3}{8} \left( \frac{R^*}{t} \right)^2 \left[ 3 + v^* \right] \left[ 1 - \left( \frac{r}{R^*} \right)^2 \right] \left[ \Delta p \right] \hspace{1cm} (9)$$

$$\sigma_\theta = \pm \frac{3}{8} \left( \frac{R^*}{t} \right)^2 \left[ 3 + v^* \right] \left[ 1 + 3v^* \right] \left( \frac{r}{R^*} \right)^2 \left[ \Delta p \right] \hspace{1cm} (10)$$

(b) Edge displacement of midplane at $r = R^*$:

$$w = \frac{VR^*}{E} \left( \frac{p_1 + p_2}{2} \right) \hspace{1cm} (11)$$

$$g = \frac{3}{2} \left( \frac{R^*}{E^* t^3} \right)^2 \left[ 1 - v^* \right] \left[ \Delta p \right] \hspace{1cm} (12)$$

A-8132.4 Pressure in Tubes or Perforations (See Figure A-8132.4-1).

(a) Stresses at any location in the equivalent solid plate:

$$\sigma_r = \sigma_\theta = h \left[ \frac{p}{h} - \frac{1}{2} \left( \frac{E^*}{E} \right) t \right] p_i \hspace{1cm} (13)$$

Uppercase "P"

(b) Edge displacements of midplane at $r = R^*$:

$$w = \frac{VR^*}{E^* \left( \frac{1}{E^*} - 1 \right) \left( \frac{1}{E^*} - 1 \right) p_i \left( \frac{2E_i}{E} t \right) \frac{h}{p} \hspace{1cm} (14)$$

where $E^*/E$ and $v^*$ should be evaluated for the ligament efficiency:

$$\frac{h}{p} = \frac{h + \left( \frac{2E_i}{E} \right) t}{p} \hspace{1cm} (15)$$

using Figure A-8131-1;

$$g = 0$$
A-8140 STRESS INTENSITIES AND STRESS LIMITS FOR PERFORATED PLATES

A-8141 Equations for Stress Intensities

In A-8140 equations are given for the stress intensities in a perforated plate using the stresses determined for the equivalent solid plate.

A-8142 Typical Ligaments in a Uniform Pattern

A-8142.1 Mechanical and Pressure Loads on Circular Plates.

(a) The stress intensity based on stresses averaged across the minimum ligament width and through the thicknesses of the plate is limited according to NB-3221.1 and is computed from the larger of:

\[ S = \frac{P}{h} \left( \frac{\Delta p R^*}{t} + \frac{W}{\pi t R^*} \right)^2 + (\bar{\sigma}_r)^2 \]  \hspace{1cm} (16)

or

\[ S = \frac{1}{2} \frac{P}{h} \left( \frac{\Delta p R^*}{t} + \frac{W}{\pi t R^*} \right)^2 + (\bar{\sigma}_r)^2 + \sigma_r + \left(2p h / P\right) \]  \hspace{1cm} (17)

where only the positive root is used. The first term under the radical reflects the effect of the transverse shear stress due to the mechanical and pressure loads. It is a maximum in the outermost ligament of the perforated region, but it may be determined for any radius, larger than c, by substituting r for R in the expression. For r < c, the W/\pi t R* term should be omitted. \( \bar{\sigma}_r \) is the stress resulting from applied in-plane loading averaged through the thickness of the equivalent solid plate. It includes the stresses due to pressure in the tubes or perforations given in A-8132.4. No bending stresses are included.

(b) The stress intensity based on stresses averaged across the minimum ligament width but not through the thickness of the plate is limited according to NB-3221.1 and is computed from

\[ S = \sigma_{ave} \]  \hspace{1cm} (18)

However, supporting interactions from adjacent members may only be considered if the primary plus secondary stresses in such members are limited to 1.5S\( _m \). Effects of temperature are not included.

A-8142.2 Combined Mechanical and Thermal Effects.

(a) The range of the stress intensity based on stresses averaged across the minimum ligament width but not through the thickness of the plate is limited according to NB-3222.2 and is computed from

\[ S = K \left( \frac{P}{h} \right) \sigma_1 \]  \hspace{1cm} (19)

where

\[ K = \text{stress multiplier from Figure A-8142-1} \]

\[ \sigma_1 = \text{larger absolute value of} \sigma_r \text{or} \sigma_\theta, \text{psi (MPa)}, \text{caused by mechanical loading or structural interaction with adjacent members, computed as the sum of the surface stresses in the equivalent solid plate using the applicable equations in A-8130 and A-8150} \]

The effects of temperature are included in the consideration of the structural interaction with adjacent members.
(b) The peak stress intensity due to all loadings is limited by cumulative fatigue considerations as described in NB-3222.4 and is given by

\[ S = Y_{\text{max}} \left( \frac{P}{h} \right) \sigma_1 + p \]

(20)

where

- \( p_s \) = pressure on the surface where the stress is being computed, psi (MPa)
- \( Y_{\text{max}} \) = stress multiplier given in Figure A-8142-2 as a function of the biaxiality ratio \( \beta = \frac{\sigma_2}{\sigma_1} \)
- \( \sigma_1 \) = principal stress being the largest absolute value in the plane of the equivalent solid plate, psi (MPa)
- \( \sigma_2 \) = principal stress having the smallest absolute value in the plane of the equivalent solid plate, psi (MPa)

(Equivalent solid plate stresses due to various loads shall be superimposed in order to obtain \( \sigma_1 \) and \( \sigma_2 \) before any multipliers are applied, and the signs of \( \sigma_1 \) and \( \sigma_2 \) should be maintained.)

The solid curves in Figure A-8142-2 give the maximum stress multipliers for the worst angular orientation of \( \sigma_1 \) and \( \sigma_2 \) with respect to the axes of symmetry \( x \) and \( y \) of the hole pattern. In some cases, the worst orientation may not exist anywhere in the plate, and the use of lower stress multipliers is justified. An important case concerns

the thermal stress produced by a temperature gradient across the diametral lane in a perforated plate. Such a gradient causes a uniaxial stress oriented parallel to the diametral lane. If the diametral lane is parallel to the \( y \) axis as shown in Figure A-8142-3, the stress multiplier given by the dashed line in Figure A-8142-2 may be used.

(c) Equation (b)(20) will give the maximum stress intensity for any loading system. Equation (b)(20) is not adequate for more complex cyclic histories where the angular orientation of the maximum stress intensity varies during the cycle. In such cases, it is necessary to compute the stress history at each angular orientation \( \phi \) using eq. (21)

\[ S_\phi = Y_1 \left( \frac{P}{h} \right) \sigma_1 + Y_2 \left( \frac{P}{h} \right) \sigma_2 + p \]

(21)

where

- \( S_\phi \) = peak stress intensity at the angular orientation \( \phi \)
- \( Y_1, Y_2 \) = stress multipliers in Figures A-8142-3 through A-8142-5 for various orientations of the principal stresses \( \sigma_1 \) and \( \sigma_2 \) computed for the equivalent solid plate

Note that these figures give stress multipliers for particular angular orientation only. The graph for the angular orientation closest to the actual angular orientation should be used. This is sufficiently accurate since the maximum possible difference between the actual orientation and the nearest orientation given in Figures A-8142-3 through A-8142-5 is only 7.5 deg. Examples for the computation of \( S_\phi \) are given as follows.

**Example 1**

The combined stresses in the equivalent solid plate for a perforated plate of 0.05 ligament efficiency were computed at a point as:

\[
\sigma_r = 750 \text{ psi} \\
\sigma_\theta = -1100 \text{ psi}
\]

and \( \sigma_r \) is rotated 12 deg, measured from the \( y \) axis of the hole pattern. To determine the value of \( \sigma_\phi \) at 40 deg from the \( y \) axis, use the following procedure: let \( \sigma_1 = \sigma_r \), \( \sigma_2 = \sigma_\theta \). Since the angular orientation of 12 deg is closest to 15 deg, use Figure A-8142-5 for the stress multipliers.

Read at \( \phi = 40 \deg \) on Scale A: \( Y_1 = +1.65 \); on Scale B: \( Y_2 = -0.70 \). Then from eq. (21), the peak stress intensity is computed as

\[
S_\phi = \frac{1}{0.05} \left( +1.65 \right) \left( 750 \right) + \frac{1}{0.05} \left( -0.70 \right) \left( -1100 \right)
\]

\[= 40,200 \text{ psi} \]
Figure A-8142-3

\[ \sigma_\phi = \frac{P}{h} Y_1 \sigma_1 \]

Stress Multiplier, \( Y_1 \)

- \( h/P = 0.05 \)
- \( h/P = 0.10 \)
- \( h/P = 0.15 \)
- \( h/P = 0.25 \)
- \( h/P = 0.33 \)
- \( h/P = 0.50 \)

\( \pm \phi \), deg.

0 10 20 30 40 50 60 70 80 90

0 0.5 1.0 1.5 2.0 2.5 3.0

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Example 2

For the same plate as above at another point, the direction of \( \sigma_r \) coincides with the \( x \) axis. Let \( \sigma_r = \sigma_2, \sigma_\theta = \sigma_1 \). Read at \( \phi = 40 \) deg from Figure A-8142-3, \( Y_1 = +2.75 \) and from Figure A-8142-4, \( Y_2 = -1.80 \). Then

\[
S_{ph} = \frac{1}{0.05}(2.75)[-1100] + \frac{1}{0.05}(-1.80)(750)
\]

\[
= -87,500 \text{ psi}
\]

**(d)** The peak stress intensity at the outermost hole is computed from

\[
S = K_r \sigma_{rim} + p_s
\]

where

\( K_r \) = a stress multiplier from Figure A-8142-6

\( \sigma_{rim} \) = the nominal circumferential stress in the rim, psi

The stresses given by eqs. (b)(20) and (c)(21) and by eq. (22) are limited by cumulative fatigue considerations, as described in NB-3222.4.

### A-8143 Irregular Ligament Patterns or Thin Ligaments in a Nominally Uniform Pattern

For irregular ligament patterns or thin ligaments in a nominally uniform pattern, the stresses are determined as given in the following subparagraphs.

**A-8143.1 Average Stress Intensity.** The stress intensity based upon the ligament stresses averaged across the ligament width and through the plate thickness due to pressure plus other mechanical loads is limited to 3.0 \( S_m \) in accordance with Table NB-3217-1. The appropriate value is computed according to **A-8142.1(a)**, where \( h_\theta \) (the actual width of the thin ligament) is used in place of the nominal width \( h \).

**A-8143.2 Peak Stress Intensity.** The peak stress intensity in the thin ligament due to mechanical loading and structural interaction with adjacent members, including thermal effects, is limited by cumulative fatigue considerations. This peak stress intensity is computed by multiplying the peak stress intensity for a nominal thickness ligament by the \( K_m \) value given in Figure A-8143.2.1.

**(a)** The peak stress intensity in nominal ligament is calculated as indicated in **A-8142.2(b)**.

**A-8150 THERMAL SKIN EFFECT

**A-8151 General Considerations

In certain cases, the temperature gradient through the thickness of a perforated plate can be closely approximated by a step change in the metal temperature near the surface of the plate. In such a case, significant thermal stresses develop only in the skin layer of the plate at the surface where the temperature change occurs and the thermal stresses in the remainder of the plate are negligible.

**A-8152 Maximum Thermal Skin Stress

The maximum thermal skin stress on the surface of a perforated plate can be computed from the relation:

\[
\sigma_{skin} = K_{skin} Y_{max} \left( \frac{P}{h} \right) \left( \frac{E \alpha}{1 - \nu} \right) (T_m - T_s)
\]

where

- \( E, \alpha, \nu \) = modified material properties
- \( h \) = ligament width
- \( P \) = pitch, in. (mm)
- \( T_m \) = mean temperature of the plate
- \( T_s \) = temperature of the plate at the surface under consideration
- \( Y_{max} \) = stress multiplier from Figure A-8142-2, for \( \beta = +1 \)

**A-8153 Peak Stress Intensities When Thermal Skin Stresses Are Included

**(a)** When thermal skin stresses are to be combined with other stresses to obtain the peak stress intensity, eq. A-8152(23) may not be used. In such a case the thermal stresses at any location on the surface of the equivalent solid plate are given by:

\[
\sigma_r = \sigma_\theta = K_{skin} \frac{E \alpha}{1 - \nu} (T_m - T_s)
\]

where

- \( E, \nu \) = unmodified material properties (since \( K_{skin} \) includes the consideration for \( E^* \) and \( \nu^* \))
- \( K_{skin} \) = stress ratio from Figure A-8153-1
- \( T_m \) = mean temperature of the plate
- \( T_s \) = surface temperature of the plate

**(b)** The equivalent solid plate stresses given by eq. A-8152(23) can then be combined with other solid plate stresses and the method given in A-8142.2(b) can be used to obtain the peak stress intensity.
This page is background information

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Contents Under Appendix I revise the following Articles to read:

Article 1-5 Analysis of Flat Circular Heads
1-510 Nomenclature
1-520 Pressure and Edge Loads on Circular Flat Plates
1-530 Flat Plate Pressure Vessel Heads
1-540 Geometry Constants
1-550 Stress Intensities in a Flat Plate

Article 1-7 Discontinuity Stresses
1-700 General
1-710 Information Required
1-720 Method of Analysis
1-730 Example Illustrating the Application of Paragraph 1-721

Article 1-9 Stresses in Ligaments
1-900 Perforated Flat Plates
1-910 Nomenclature.
1-920 Analysis of Circular Perforated Area
1-930 Stress Intensities and Stress Limits for Perforated Plates
1-940 Thermal Skin Effect

Par. N-331.1 In the second paragraph, revise reference to E 23-60 to read: E23-64

Table N-421 Add specifications SA-479, SA-515 and SA-516 and their design stress intensity values shown in this Addenda.

Table N-424 Add specifications SA-479, SA-515 and SA-516 and their values of yield strength shown in this Addenda.

Fig. N-462.3 Revise the note in Fig. N-462.3 which defines \( t_w \) to read: \( t_{w \ min} = \) the lesser of \( t_n \) or \( t/2 \).

Par. N-469 Delete this paragraph.

Par. N-518.1 At end of paragraph, add a new sentence to read: However, the configuration shown by (c), (f), and Section B-B shall not be used in vessels which require analysis for cyclic operation.

Par. N-533 (d) Delete the last sentence in this paragraph.

Article 1-5 On Page 102, delete reference to Article 1-5 and add an Article 1-5 to read:

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ARTICLE I-9

Stresses In Ligaments

I-900 PERFORATED FLAT PLATES

This article contains a method of analysis for flat perforated plates when subjected to directly applied loads or loadings resulting from structural interaction with adjacent members. This method applies to perforated plates which satisfy the following conditions:

(a) The holes are in an array of equilateral triangles.
(b) The holes are circular.
(c) There are nineteen or more holes.
(d) The ligament efficiency is greater than 5 per cent ($\eta \geq 0.05$)
(e) The plate is thicker than twice the hole pitch ($t/P \geq 2$). If only inplane loads and/or thermal skin stresses are considered this limitation does not apply.

Credit may be taken for the stiffening effect of the tubes in the perforations. The extent to which the tubes stiffen the perforated plate depends on the materials, the manufacturing processes, operating conditions and degree of corrosion. This stiffening effect may be included in the calculations by including part or all of the tube walls in the ligament efficiency used to obtain the effective elastic constants of the plate. Such stiffening may either increase or decrease stresses in the plate itself and in the attached shells.

Credit may be taken for the staying action of the tubes where applicable.

I-910 NOMENCLATURE

(1) $h$ = Nominal width of ligament at the minimum cross section.
(2) $P$ = Nominal distance between hole centerlines (pitch).
(3) $t$ = Thickness of plate exclusive of cladding or corrosion allowance.
(4) $\eta$ = Ligament efficiency = $\left( \frac{h}{P} \right)$
(5) $\nu^*$ = Effective Poisson's ratio for perforated plate (see Figure I-920).
(6) $E^*$ = Effective Young's modulus for perforated plate (see Figure I-920).

(7) $R^*$ = $r_0 + 1/4 (P - h)$; the effective radius of the perforated plate.
(8) $r_0$ = Radial distance from center of plate to center of outermost hole.
(9) $p_1$, $p_2$ = Pressures acting on surfaces of the plate.
(10) $\Delta p$ = Differential pressure across the plate.
(11) $\sigma_r$ = Radial stresses in the equivalent solid plate.
(12) $\sigma_\theta$ = Tangential stress in the equivalent solid plate.
(13) $\beta$ = Biaxiality ratio = $\left( \frac{\sigma_r}{\sigma_\theta} \right)$
\[
\left( \frac{\sigma_r}{\sigma_\theta} \right) \text{ where } -1 \leq \beta \leq 1.
\]
(14) $\sigma_{ave}$ = Larger absolute value of $\sigma_r$ or $\sigma_\theta$. (See Par. I-931.1(2).)
(15) $\sigma_r$ = Radial stress averaged through the depth of the equivalent solid plate.
(16) $\sigma_1$, $\sigma_2$ = Principal stress in the plane of the equivalent solid plate. (See Par. I-931.2.)
(17) $\sigma_{rim}$ = Nominal circumferential stress in solid rim.
(18) $\sigma_{skin}$ = Thermal skin stress.
(19) $r$ = Designation of radial location in plate.
(20) $\rho$ = Radius of holes in the plate.
(21) $K$ = Stress multiplier for stresses averaged across the width of the ligament but not through the thickness (see Figure I-930a).
(22) $Y$ = Stress multiplier for peak ligament stresses (See Figure I-930b.)
(23) $K_{m}$ = Ratio of peak stress in reduced ligament to the peak stress in normal ligament.
(24) $K_{skin}$ = Stress multiplier for thermal skin stress (see Figure I-940).
I-931.2 Combined Mechanical and Thermal Effects

(1) The range of the stress intensity based on stresses averaged across the minimum ligament width but not through the thickness of the plate is limited according to N-414.4 and is computed from

\[ S = K \frac{P}{h} \sigma_1 \]  

(a)

where:

- \( K \) is the stress multiplier from Figure I-930(a).
- \( \sigma_1 \) is the larger absolute value of \( \sigma_r \) or \( \sigma_\beta \), caused by mechanical loading or structural interaction with adjacent members, computed as the sum of the surface stresses in the equivalent solid plate using the applicable formulas in paragraph I-920 and I-940. The effects of temperature are included in the consideration of the structural interaction with adjacent members.

(2) The peak stress intensity due to all loadings is limited by cumulative fatigue considerations as described in N-415 and is given by:

\[ S = Y_{max} \frac{P}{h} \sigma_1 + p_x \]  

(a)

where:

- \( p_x \) is the pressure on the surface where the stress is being computed.
- \( \sigma_1 \) is the principal stress having the larger absolute value in the plane of the equivalent solid plate.
- \( \sigma_2 \) is the principal stress having the smaller absolute value in the plane of the equivalent solid plate.

Equivalent solid plate stresses due to various loads should be superimposed in order to obtain \( \sigma_1 \) and \( \sigma_2 \) before any multipliers are applied, and the signs of \( \sigma_1 \) and \( \sigma_2 \) should be maintained.

\( Y_{max} \) is the stress multiplier given in Figure I-930(b) as a function of the biaxiality \( \beta = \frac{\sigma_2}{\sigma_1} \).

The solid curves in Figure I-930(b) give the maximum stress multipliers for the “worst" angular orientation of \( \sigma_1 \) and \( \sigma_2 \) with respect to the axes of symmetry, \( x \) and \( y \), of the hole pattern. In some cases, the “worst" orientation may not exist anywhere in the plate, and the use of lower stress multipliers is justified. An important case concerns the thermal stress produced by a temperature gradient across the diametral lane in a perforated plate. Such a gradient causes a uniaxial stress oriented parallel to the diametral lane. If the diametral lane is parallel to the \( y \) axis as shown in Figure I-930(c), the stress multiplier given by the dashed line in Figure I-930(b) may be used.

Equation (a) will give the maximum stress intensity for any loading system. Equation (a) is not adequate for more complex cyclic histories where the angular orientation of the maximum stress intensity varies during the cycle. In such cases, it is necessary to compute the stress history at each angular orientation \( \phi \) using equation (b):

\[ S_\phi = Y_1 \frac{P}{h} \sigma_1 + Y_2 \frac{P}{h} \sigma_2 + p_x \]  

(b)

where:

- \( S_\phi \) is the peak stress intensity at the angular orientation \( \phi \).
- \( Y_1 \) and \( Y_2 \) are the stress multipliers given in Figures I-930(c), (d), and (e) for various orientations of the principle stresses \( \sigma_1 \) and \( \sigma_2 \) computed for the equivalent solid plate. Note that these figures give stress multipliers for particular angular orientations only. The graph for the angular orientation closest to the actual angular orientation should be used. This is sufficiently accurate since the maximum possible difference between the actual orientation and the nearest orientation given in Figures I-930(c), (d), and (e) is only 7.5 degrees. Examples for the computation of \( S_\phi \) are given below:

**Example 1:**

The combined stresses in the equivalent solid plate for a perforated plate of .05 ligament efficiency were computed at a point as:

- \( \sigma_r = 750 \text{ psi} \)
- \( \sigma_\beta = -1100 \text{ psi} \)
and $\sigma_r$ is rotated 12 degrees measured from the y axis of the hole pattern. To determine the value of $\sigma_\theta$ at 40 degrees from the y axis, use the following procedure: let $\sigma_1 = \sigma_r$, $\sigma_2 = \sigma_\theta$. Since the angular orientation of 12 degrees is closest to 15 degrees, use Figure I-930(e) for the stress multipliers. Read at $\phi = 40^\circ$ on Scale A: $Y_1 = +1.65$; on Scale B: $Y_2 = -0.70$. Then from I-931.2 (2) (b) the peak stress intensity is computed as

$$S_\phi = \frac{1}{.05} (1.65) (750) + \frac{1}{.05} (-0.70)$$

$$(-1100) = 40,200 \text{ psi}$$

**Example 2.**

For the same plate as above, at another point, the direction of $\sigma_1$ coincides with the x axis. Let $\sigma_1 = \sigma_2$, $\sigma_\theta = \sigma_r$. Read at $\phi = 40^\circ$ from Figure I-930(c) $Y_1 = +2.75$ and from Figure I-930(d) $Y_2 = -1.80$. Then:

$$S_\phi = \frac{1}{.05} (2.75) (-1100) + \frac{1}{.05} (-1.80)$$

$$(-750) = -87,500 \text{ psi}$$

(3) The peak stress intensity at the outermost hole is computed from

$$S = K_r \sigma_{rim} + p_x$$

where:

- $K_r$ is a stress multiplier from Figure I-930(f).
- $\sigma_{rim}$ is the nominal circumferential stress in the rim.

The stresses given by equations (2) (a), (2) (b) and (3) (a) are limited by cumulative fatigue consideration as described in N-415, Analysis for Cyclic Operation.

**I-932 Isolated or Thin Ligament**

In an irregular or thin ligament in a nominally uniform pattern the stresses are determined as follows:

$1-932.1$ The stress intensity based upon the ligament stresses averaged across the ligament width and through the plate thickness due to pressure plus other mechanical loads is limited to 3.0 $\bar{Sm}$ in accordance with Table N-413. The appropriate value is computed according to paragraph I-931.1(1) where $h_0$ (the actual width of the thin ligament) is used in place of the nominal width $h$.

$1-932.2$ The peak stress intensity in the thin ligament due to mechanical loading and structural interaction with adjacent members, including thermal effects, is limited by cumulative fatigue considerations. This peak stress intensity is computed by multiplying the peak stress intensity for a nominal thickness ligament by the $K_m$ value given in Figure I-930(g).

The peak stress intensity in a nominal ligament is calculated as indicated in paragraph I-931.2(2).

**I-940 THERMAL SKIN EFFECT**

In certain cases, the temperature gradient through the thickness of a perforated plate can be closely approximated by a step change in the metal temperature near the surface of the plate. In such a case, significant thermal stresses develop only in the skin layer of the plate at the surface where the temperature change occurs and the thermal stresses in the remainder of the plate are negligible.

The maximum thermal skin stress on the surface of a perforated plate can be computed from the relation:

$$q_{skin} = K_{skin} Y_{max} \left( \frac{2}{h} \right) \left( \frac{E\alpha}{1 - \nu} \right) \left( T_m - T_s \right)$$

where:

- $E$, $\alpha$, $\nu$ are the unmodified material constants.
- $Y_{max}$ is the stress multiplier from Figure I-930(b), for $\beta = +1$, $T_m$ is the mean temperature of the plate.
- $T_s$ is the temperature of the plate at the surface under consideration.
- $P$ and $h$ are the pitch and ligament width respectively.

When thermal skin stresses are to be combined with other stresses to obtain the peak stress intensity, equation (a) above may not be used.
In such a case the thermal stresses at any location on the surface of the equivalent solid plate are given by:

\[
\sigma_t = \sigma_0 = K_{\text{skin}} \frac{E\alpha}{1 - \nu} (T_m - T_s) \quad \text{(b)}
\]

where:

- \( K_{\text{skin}} \) = A stress ratio from Figure I-940,
- \( \sigma_t = \sigma_0 \) = The mean temperature of the plate,
- \( T_m \) = The surface temperature of the plate,
- \( E, \alpha \) = The material constants, unmodified.

The equivalent solid plate stresses given by equation (b) can then be combined with other solid plate stresses and the method given in paragraph I-931.2(2) can be used to obtain the peak stress intensity.