Self-similarity as a tool for verification

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Abstract

For most “real” problems of interest in computational physics and fluid dynamics (i.e. with complex geometry, initial and boundary conditions, and frequently with multiple physics couplings), there are no known analytic or semi-analytic solutions.

Indeed, this is also true for many simplified problem definitions which are very easy to characterize (single physics, simple initial and boundary conditions) – an example being the set of two-dimensional hydrodynamic Riemann problems. Such cases may often appear as sub-components of more complex systems, and so understanding them can be highly useful to gain leverage on understanding the larger system.

However, while fully analytic solutions remain elusive, many such problems can be shown to be self-similar under a particular coordinate transformation. This self-similarity is a type of symmetry of the underlying equations, and are characterized by the fact that under the appropriate transformation of variables the solution is unchanging (e.g., no inherent length- or time-scales). However, upon discretization of the governing equations, length and time scales associated with the mesh and time-step size are introduced which seed numerical error.

Consequentially, in the self-similarly transformed frame, the solution at “early” times will be essentially very poorly resolved while at late times it effectively becomes more refined.

We present several example problems, including the triple-point problem and various oblique shocks, where we use this behavior as a verification tool to assess solution quality and numerical error with respect to simulation time and effective computational cost (i.e., number of compute cycles).
Outline

• Intro – motivating examples: manufacturing, diagrams, and curved HE expts

• Hydrodynamic self-similarity (BRIEFLY)

• 1D Riemann problem statement and solution

• 2D Riemann problem statement and “solution” (Triple-point problem)

• Oblique Shock problem statement and solution (also 2D Riemann problem)

• Back to motivating examples and possible extensions, like shaped charges and EOS experiments.
Introduction – machining, diagrams, experiments

planar shockwave

some possible manufacturing defects

A relevant simulation we’re asked to “VVUQ”.

PDV Probes
Steel 304L Cylindrical Shell
Tin Elliptical Shell
PBX-9502
LWG

Time = 16.0 μsec

Incoming Planar Shock

Ambient Air

Steel Wedges

θ_w1

θ_w2
1D Riemann prob. statement, solution and self-similarity

- 1D Riemann problems, such as the Sod shock tube, can be solved analytically using self-similar coordinates, as done in ExactPack.
Simplest 2D Riemann problem: the triple-point problem (no known analytic solutions)

A 2\textsuperscript{nd}-order code solving a shock-physics problem should show 1st-order accuracy.
Simplest 2D Riemann problem, in self-similar coordinates

- Solution verification and convergence analysis (comparing a single simulation to itself)

The solution appears to refine itself in “time”.

Simplest 2D Riemann problem (Triple-point)

- Solution verification comparing a single simulation to its last time, to show 1st-order accuracy.

![Graph showing AMR results and non-AMR results over simulated time.](image)
The Oblique Shock, another 2D Riemann problem

- Different shock structures produced by changing shock velocity or wedge angle.
- Exact solutions exist for some portions of the domain via shock polars, and “70%” solutions (“not obviously wrong”) determined by self-similarity.

$$p_1 = 4.5$$  
$$\rho_1 = 0.0026667$$  
see shock polar diagram

$$p_2 = 14.24$$  
$$\rho_2 = 0.0058$$  
see shock polar diagram

$$p_2 = 14.35136771...$$

$$\rho_2 = 0.005843141...$$

$$p_1 = 4.5$$

$$\rho_1 = 0.002666...$$

### Exact solution in ExactPack

**M0 = 2 shock polar diagram**
Oblique Shock

Physical coordinates

Self-similar coordinates

Diff w.r.t. Final time

Self-similar coordinates

Rel. diff w.r.t. Final time
Oblique Shocks for solution verification and convergence analysis by comparing earlier times to hi-res final time.
Self-similarity as a tool for verification

• Self-similar simulations resolve themselves in time.

• In time, we assert they resolve at the correct rate.

• This allows for a solution verification method requiring only a single simulation.

• The latest time is the fiducial solution for the solution verification analysis.

• Failure to preserve self-similarity implies the code introduces anomalous length or time scales.

• Self-similarity is a symmetry of hydro, diffusion, and other physics models.
Other examples: HE applications and shaped charges

Possibly as a way of accessing high-pressure EOS regions, or possibly testing EOS constructions.
Self-similarity as a tool for verification

• Thanks for your time!

• Questions?

• Recent results in 1D confirm our multi-D assertions.

• I think MMS of the self-similar equations used in physical (x,t) space should allow for interesting results!

• Contact Jim Ferguson (jmferguson@lanl.gov) for more information.