Sensitivity Analysis of a Nuclear Reactor System
Finite Element Model

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Outline

- Overview of System Finite Element Modeling for Nuclear Reactors
- Motivation for Sensitivity Analysis
- Single-factor Sensitivity Analysis Results
- Global Sensitivity Analysis Methodology
Motivation

“This plant—using the power of the atom to supply electrical power—represents what can be done, not only in America, but throughout the world, to put the atom to work for the good of mankind.”

— PRESIDENT DWIGHT D. EISENHOWER

National Security
America’s nuclear energy industry shapes international safety rules. But our leadership is now at risk. Learn why nuclear energy is mission-critical to our national security.

Climate
Nuclear produces more of America’s emission-free electricity than all other clean energy sources combined. See how nuclear fights climate change on multiple fronts.

Infrastructure
Americans will need 33 percent more electricity by 2050. Our current grid can’t keep pace. Learn how nuclear power can close the gap, carbon-free.

Clean & Reliable

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Overview of Reactor System
Overview of Reactor Internals

- Reactor Vessel (RV)
- Upper Internals (UI)
  - Guide Tube Assembly
    - Lower Guide Tubes (LGT)
    - Upper Guide Tubes (UGT)
  - Upper Support Columns (USC)
- Lower Internals (LI)
  - Core Barrel (CB)
  - Core Shroud (CS)
  - Secondary Core Support Structure (SCSS)
Why a System Finite Element Model?

- Based on Westinghouse legacy and current experience system models are necessary for accurate dynamic predictions and system-level phenomenological understanding
  - A significant amount of energy is imparted to the system during faulted conditions
    - Seismic excitation (base motion) occurs at the reactor vessel supports
    - Loss-of-coolant accident (LOCA) propagates a pressure wave from a piping break through the reactor vessel
- The RVI system response is characterized by components with complex interconnected boundaries
  - Both fluid and structural
- System model is used to characterize the response of the internals and generate loadings for downstream sub-system analyses
- Necessary for plant qualification during faulted conditions
System Finite Element Model Development
for Generation 3+ Nuclear Plants

- SFEM is composed of linear and non-linear features designed to capture RVI response
- Water is represented as hydrodynamic mass to account for effects on modal frequencies and dynamic response
- Hydrodynamic mass in the Lower Internals (LI) is established in accordance with Fritz’s approximation of the effect of liquids on the dynamic motions of immersed solids
  - LI consist of two cylindrical annuli
  - Each annuli is segmented in the SFEM and subsequently connected by a mass matrix at multiple elevations expressed as a Fourier series
Dynamic Response of Reactor System

- Dynamics considered associated with non-stationary stochastic processes
  1. Loss of Coolant Accident (LOCA), shown below, and
  2. Seismic

Maximum Accel. over time

Dominant modes

Response may be characterized in Time or Frequency Domain

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Some Questions

- How do I ensure the structural **safety** of nuclear reactor components while simultaneously:
  - Maintaining conservative design margins, and
  - Keep engineering cost to an acceptable level
- What is **important** and what is unimportant to the design and analysis of a reactor assembly?
- How do I **verify** that my evaluation of structural safety did not neglect something important?
- But I’m **unsure** about much of this

**Challenge:** Intelligently ensure **safety** and **minimize costs** amidst significant **uncertainties**

**Hypothesis** = Sensitivity analysis which accounts for parameter uncertainties provides highly valuable insight as to the changes in dynamic behavior of the reactor system, which serves to preclude potentially unnecessary (and costly) engineering evaluations. This serves to provide confidence in **safety** while minimizing **costs** in the context of non-negligible **uncertainty**.
Parameterization of Inputs

A variety of types of model parameters were varied in different locations throughout the reactor system.

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Parameter</th>
<th>Location</th>
</tr>
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<tbody>
<tr>
<td>$k(\omega)_1$</td>
<td>Rotational Stiffness</td>
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<tr>
<td>$k(y)_1$</td>
<td>Vertical Stiffness</td>
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</tr>
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<td>$m_2$</td>
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<td>Radial Stiffness</td>
<td>3</td>
</tr>
<tr>
<td>$g(\theta)_3$</td>
<td>Circumferential Gap</td>
<td>3</td>
</tr>
<tr>
<td>$g(r)_3$</td>
<td>Radial Gap</td>
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</tr>
<tr>
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<td>Circumferential Gap</td>
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<td>$g(r)_4$</td>
<td>Radial Gap</td>
<td>4</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Damping</td>
<td>N/A</td>
</tr>
<tr>
<td>$g(y)_5$</td>
<td>Vertical Gap</td>
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<tr>
<td>$N_6$</td>
<td>Number of Components</td>
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</tr>
<tr>
<td>$k_7$</td>
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<td>$k_9$</td>
<td>Stiffness</td>
<td>9</td>
</tr>
<tr>
<td>$k_{10}$</td>
<td>Stiffness</td>
<td>10</td>
</tr>
</tbody>
</table>

Each model parameter variation defined according to perceived uncertainties (combination of aleatory and epistemic)
Single-Factor Sensitivity of Maximum Response over Time

- As one model parameter is varied by ±50%, the change in the maximum response is plotted for 6 different locations.
- This provides a basic understanding of non-linearities and trending of certain outputs when a particular input value is in question.

Trend of model response is complicated when parameters vary.
Single-Factor Sensitivity of Acceleration Response Spectra

- Effect on dynamic response of changing one parameter across its range of variation

Visualization for Variation in Dynamic Response

Variation from Nominal Value of Input Param.

Acceleration (G)
Method for Global Sensitivity Analysis (GSA)

1. Construct computational design of experiment
2. Train and verify Surrogate model
3. Refine Surrogate model by adaptive sampling
4. Compute Global sensitivities

Implemented with ANSYS DesignXplorer, with basic methodological precedence for surrogate-based GSA in literature such as Gratiet, Marelli, & Sudret (ETH-Zurich)
1 – Computational Design of Experiment

- Latin Hypercube Sampling (LHS) used as a means of randomly sampling the parameter space
  - Evaluated Optimal Space Filling (OSF) design and determined that impact on sampling requirements for a given Surrogate error were not significant
- It is recognized that advanced sampling methods such as proposed by Shields (e.g., Latinized Partially Stratified Sampling) could have provided more substantial benefit, but that was not explored as part of this work
2 – Training and Verification of Surrogate Model

• A unique genetic aggregation surrogate model was trained from the computational DOE
  – Similar to “Multi-model Kriging”

1. Start with candidate surrogates (i.e., Kriging)

\[ M^K(x) = \beta^T f(x) + \sigma^2 Z(x, \xi) \]

2. Cross-validation based error calculation

\[ \text{PRESS}_{RMSE}(\hat{y}_{\text{ens}}) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( y(x_i) - \hat{y}_{\text{ens},-i}(x_i) \right)^2} \]

3. Iterate with Genetic Algorithm to determine weights

\[ \hat{e}_{\text{ens},-i}(x) = \sum_{j=1}^{NM} w_j \bar{e}_{j,-i}(x) \]

\[ c_{ij} = \frac{1}{N} E_i E_j \quad \rightarrow \quad w = \frac{C^{-1}[I]}{[I]^T C^{-1}[I]} \]

4. Obtain aggregate ensemble of surrogates

\[ \hat{y}_{\text{ens}}(x) = \sum_{j=1}^{NM} w_j \hat{y}_j(x) \]
2 – Training and Verification of Surrogate Model

- Multiple criteria used to quantify the Surrogate “goodness of fit”
- Two metrics focused upon in order to characterize error from the standpoint of:
  - maximum overall
  - averaged
- Quantified with respect to both:
  - Independent verification points
  - Cross-validation (i.e., leave-one-out)

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of Determination ($R^2$)</td>
<td>$1 - \frac{\sum_{i=1}^{N} (y_i - \hat{y}<em>i)^2}{\sum</em>{i=1}^{N} (y_i - \bar{y})^2}$</td>
</tr>
<tr>
<td>Maximum Relative Residual</td>
<td>$\max_{i=1:N} \left( \frac{</td>
</tr>
<tr>
<td>Root Mean Square Error</td>
<td>$\sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}$</td>
</tr>
<tr>
<td>Relative Root Mean Square Error</td>
<td>$\sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{y_i - \hat{y}_i}{y_i} \right)^2}$</td>
</tr>
<tr>
<td>Relative Maximum Absolute Error</td>
<td>$\frac{1}{\sigma_y} \max_{i=1:N} (</td>
</tr>
<tr>
<td>Relative Average Absolute Error</td>
<td>$\frac{1}{\sigma_y} \frac{1}{N} \sum_{i=1}^{N}</td>
</tr>
</tbody>
</table>

$y_i$ Value of the output parameter at the $i^{th}$ sampling point
$\hat{y}_i$ Value of the Surrogate model at the $i^{th}$ sampling point
$\bar{y}$ Arithmetic mean of the values of $y_i$
$\sigma_y$ Standard deviation of the values of $y_i$
$N$ Number of sampling points (i.e., “design points”)

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2 – Training and Verification of Surrogate Model

- Surrogate model error quantified with respect to both:
  - Independent verification points
  - Cross-validation (i.e., leave-one-out)
- The correlation between verification point and cross-validation point root mean squared error (RMSE) for ~25 model outputs
  - Similar observations across ~15 different plant SFEMs

Potentially minimizes requirements for independent verification samples

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3 – Surrogate refinement with Adaptive Sampling

Based on the Surrogate verification error metrics, additional sampling points may be defined adaptively by estimating the location within the parameter space for which error is greatest (and would thus be minimized by additional sampling points).

This is done using a Universal Prediction (UP) probability distribution.

1. Establish weights based on the distance between the $i^{th}$ design point and the sampled input parameter value

$$w_{i,n}(x) = \frac{1 - e^{-\frac{d(x,x_i)^2}{\rho^2}}}{\sum_{j=1}^{n} \left( 1 - e^{-\frac{d(x,x_j)^2}{\rho^2}} \right)}$$

2. Define UP-distribution as the empirical distribution of all the predictions provided by cross-validation sub-models weighted by local smoothed masses.

$$\mu_{(n,x)}(dy) = \sum_{i=1}^{n} w_{i,n}(x) \delta_{s_{n-i}(x)}(dy)$$

$$\hat{m}_n(x) = \int y \mu_{(n,x)}(dy) = \sum_{i=1}^{n} w_{i,n}(x) \hat{s}_{n-i}(x)$$

$$\hat{\sigma}^2_n(x) = \int (y - \hat{m}_n(x))^2 \mu_{(n,x)}(dy) = \sum_{i=1}^{n} w_{i,n}(x) (\hat{s}_{n-i}(x) - \hat{m}_n(x))^2$$

3. Define local UP mean and variance.

4. Select parameter value for next sampling point.

$$x_{n+1} \in \arg\max_{x \in \mathcal{X}} \gamma_n(x)$$

$$\gamma_n(x) = \hat{\sigma}^2_n(x) + \delta_d x_n(x)$$
Summary of 2 & 3 – Surrogate Training + Adaptive Refinement

• The aggregate surrogate model is the weighted sum of \( k \) surrogate models

\[
\hat{y}_{\text{ens}}(x) = w_{a(1)} \times \hat{y}_1(x) + \cdots + w_{a(k)} \times \hat{y}_k(x)
\]

• For a given point \( x \), the UP uncertainty estimation of a surrogate model is based on the weighted sum of \( N \) cross-validation sub-models

\[
\hat{m}(x) = w_{b(1)}(x) \times \hat{y}_{-1}(x) + \cdots + w_{b(N)}(x) \times \hat{y}_{-N}(x)
\]

• For the aggregate surrogate model, a cross-validation sub-model corresponds to an aggregation of cross-validation sub-models, such as:

\[
\begin{align*}
\hat{y}_{\text{ens}-1}(x) &= w_{a(1)} \times \hat{y}_{-1,1}(x) + \cdots + w_{a(k)} \times \hat{y}_{-1,k}(x) \\
\vdots \\
\hat{y}_{\text{ens}-N}(x) &= w_{a(1)} \times \hat{y}_{-N,1}(x) + \cdots + w_{a(k)} \times \hat{y}_{-N,k}(x)
\end{align*}
\]

• And the UP mean may correspondingly be expressed as:

\[
\hat{m}(\hat{y}_{\text{ens}}(x)) = w_{a(1)}(x) \times \hat{m}(\hat{y}_1(x)) + \cdots + w_{a(k)}(x) \times \hat{m}(\hat{y}_k(x))
\]
4 – Compute Global Sensitivities

• As a measure of global sensitivity, a Spearman rank-order correlation coefficient provides a measure of the monotonicity of the response trending.

\[ r_s = \frac{\sum_i^n (R_i - \bar{R})(S_i - \bar{S})}{\sqrt{\sum_i^n (R_i - \bar{R})^2} \sqrt{\sum_i^n (S_i - \bar{S})^2}} \]

• It is recognized that Saltelli (and others) recommend the use of multiple global sensitivity measures to evaluate parameter importance, so the Fourier Amplitude Sensitivity Test (FAST) was also used as a variance-based means of assessing global sensitivity.

\[ S_i = \frac{V[E(\epsilon | p_i)]}{V(\epsilon)} \]
Verification of Methodology

- The following investigations provide confidence as to the veracity of the methodology employed:
  1. Changing the number of genetic algorithm iterations ("generations") used in training the aggregate surrogate model had minimal effect on surrogate accuracy beyond approximately 8 generations.
  2. From the prior slide, it may be seen that for a given model output, the influential parameters identified by Spearman’s rank correlation and FAST are equivalent.
  3. It was found that Spearman’s rank correlation results did not appreciably change, for a given number of DOE samples, whether computed from the Surrogate or from the full-order model.
  4. The SFEM output histograms did not meaningfully change when the number of DOE samples was increased by 2X or 3X.
Conclusion

• An approach using a unique approach to surrogate modeling and sensitivity analysis was found to be of practical value to reduce costs associated with engineering design of nuclear reactor components

• The surrogate model was verified with independent sampling points as well as cross-validation

• The surrogate-based sensitivity analysis method was verified by multiple independent means

• Future work may seek to consider:
  – Advanced sampling methods to reduce overall computational expense
  – Sensitivity of forcing functions as well as model parameters
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<td>Banyay Gregory A</td>
<td>May-15-2018 09:39:09</td>
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<tr>
<td>Manager Approval</td>
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