V&V Challenge Problem: An efficient Monte Carlo method incorporating the effects of model error

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Meet the Team

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Overall Process

Model Validation & Prediction under uncertainty

Experimental Data

Validation

Computational Model
Overall Process

Model Validation & Prediction under uncertainty

Experimental Data
- Error
- Uncertainty

Computational Model
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Computational Model

Error
Uncertainty
Reliability Analysis
Prediction

Validation

①
②
③
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Probabilistic Quantification

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Prediction with Uncertainty

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Prediction
Reliability Analysis
Data & Model Uncertainty

Model Validation & Prediction under uncertainty

① Calibration of Uncertain Parameters

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Computational Model

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ASME Verification & Validation Symposium
Material Characterization

- Legacy material specs. are available
  - These are taken lightly
    - No testing, tolerances, etc.
- Material testing performed on specimens from a single tanks (compute \( E, \nu, T \))
  - Samples taken from 10 locations
    - Spatial variability exists in material parameters
      - Particularly in tank wall thickness \( T \)
    - Significant **epistemic** uncertainty
      - Is this tank representative of the other tanks?
        - Tanks vary in age.
        - Were all tanks constructed at the same facility, with the same batch material, etc.?
- Very few data points
- This tank also failed
  - Is this tank weaker than others? …Probably
Material Characterization

- Test is treated as “representative” of material behavior
  - Assume Normal distribution
  - This is believed to be a conservative estimate
    - Material tests suggest this tank is weaker than specs.
      \[ \langle E \rangle = 2.8141E + 07 \text{ psi} < E_{\text{legacy}} = 3.0E + 07 \text{ psi} \]
      \[ \langle T \rangle = 0.23132 \text{ in.} < T_{\text{legacy}} = 0.25 \text{ in.} \]
    - Computational model accepts only one parameter value:
      - Spatial variability cannot be accounted for
      - Randomly drawn values are attributed to the entire tank
    - This tank failed!

- Material testing of at least one additional tank is highly recommended
Tank Dimensions

• Again, legacy specs. are provided
  – These are again taken lightly

• Tank dimensions are given for Tanks 1 & 2:
  – Data is said to be “very accurate”
  – This is useful in determining “best fit” model parameters
    • Explicitly use for pressure loading test on Tanks 1 & 2.
    • Probability model used for Tanks 3 – 6
      – Small dataset (2 tanks, 5 measurements per tank) = high epistemic uncertainty
      – Normal Distribution

• Probability model appears to be conservative
  – Both measured tanks are greater in length & radius than legacy specifications
  – Spatial variability cannot be factored into the model
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Validation

② Model validation and calibration
Construction of “Best Fit” Model

• Match a subset of the data
  – Use the remaining data as validation

• Identify the combination of model input parameters that best matches the test data:
  – $E$, $\nu$, $T$, $R$, $L$, and $P$
    • Material parameters & thickness are bounded by $\pm 3\sigma$ from test data
    • $R$, $L$ data correspond to the tanks that were tested
  – Displacement readings have HUGE uncertainty & inconsistency
    • This will be dealt with separately
Use simulated annealing to identify “best fit” parameters

1. Randomly initialize the 6 parameters
   - E, ν, T, R, L, and P

2. Run simulation and compute error
   \[ \varepsilon = \sqrt{\sum_{i=1}^{N_p} \sum_{j=1}^{N_s} \sum_{k=1}^{N_T} (d_{ijk}^{\text{sim}} - d_{ijk}^{\text{exp}})^2} \]

3. Randomly select a parameter and perturb it

4. Rerun simulation and compute error
   1. If error is improved, accept the perturbation
   2. If error is worse, reject the perturbation with 95% probability
      - Note, this is not a “variable temperature” implementation but such implementation could improve computational expense in the future
“Best Fit” Model

![Graph showing iteration numbers vs. L2 norms and pressure vs. displacement for Tank 1.](image)
Computational Expense

- Simulated Annealing can be computationally quite expensive
  - Perform this on the low resolution model
  - Need ~500 iterations
  - SA cannot be parallelized

\[ 12 \text{ CPU-hrs} \times 2 \text{ Sims/Iter} \times 500 \text{ Iter} = 12,000 \text{ hrs!!!} \]
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Parallelize

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Parallelize??

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1 \text{ hr} \times 500 \text{ Iter} = 500 \text{ hrs}
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Computational Expense

• 500 hours is still 3 full weeks of computation!
• We can further reduce computational cost
  1. Sensitivity Analysis
     • Reduces stochastic dimension
        – Sensitivity to $L$ appears to be smaller than others
  2. $E$ and $v$ are correlated (data shows $p \approx 0.7$)
     • Principal Component Analysis can reduce this to a single variable (with small error)
  3. Pressure can be prescribe rather than randomized (not a model parameter per se)
     • Uncertainty in pressure treated elsewhere
• Reduced set of 2-3 random variables

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Reliability Analysis
- Prediction

- Performed using newly developed Targeted Random Sampling Method for efficient Monte Carlo reliability analysis
  - Samples heavily in near the limit state
  - Rooted in stratified sampling design
  - Consider this a “black box” here
  - Produces a model reliability prediction

③ Computational reliability analysis
Overall Process

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Computational Model
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Validation

④ Experimental data error analysis

Prediction

Reliability Analysis
Data Error & Uncertainty

• Displacements are stated to be accurate to within ±3% or 0.002 in. – whichever is greater.
  – This is a very large range (same order of magnitude as the data itself in some cases)

• Pressure measured is within ±5% of the absolute pressure
Data Error & Uncertainty

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• Displacement data error is very large in general
• There is inconsistency with the data and the error specifications at high pressure
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What could have happened?
Data Error

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What could have happened?

- Human error
- Defective gauge
- Etc.
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This is life! What do we do?
Data Error

- Displacement data error is very large in general
- There is inconsistency with the data and the error specifications at high pressure

What could have happened?

- Human error
- Defective gauge
- Etc.

This is life! What do we do?

- Identify bad data and systematically remove it
- Recognize that our model inherits this error!
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Probabilistic Quantification
- Reliability Analysis
- Prediction

5) Quantification of model error
Quantifying Model Error

• Model error defined at location $i$ as follows:

$$\varepsilon_i = \frac{d_{i}^{\text{best}} - d_{i}^{\text{exp}}}{d_{\text{rms}}}$$

$$d_{\text{rms}} = \sqrt{\frac{1}{N_p} \sum_{i=1}^{N_p} (d_{i}^{\text{best}})^2}$$

• We construct an empirical distribution for $\varepsilon_i$ utilizing all available test data and the associated “best fit” models

• Note, $d_{i}^{\text{best}}$ is computed using the high fidelity model
  – Requires one model run for each set of test data
Model Error Transformation

• Model errors are quantified in terms of radial displacements.
• We want model error in terms of stress.

\[
\begin{align*}
    u(x, \phi) &= \sum_{m=1,3,5,...,M} \sum_{n=0,1,2,...,N} A_{mn} \cos(n\phi) \cos\left(\frac{m\pi x}{l}\right) \\
    v(x, \phi) &= \sum_{m=1,3,5,...,M} \sum_{n=0,1,2,...,N} B_{mn} \sin(n\phi) \sin\left(\frac{m\pi x}{l}\right) \\
    w(x, \phi) &= \sum_{m=1,3,5,...,M} \sum_{n=0,1,2,...,N} C_{mn} \cos(n\phi) \sin\left(\frac{m\pi x}{l}\right)
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Stochastic field perturbation
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Stochastic field perturbation

• The distribution of stress is computed from \(\tilde{w}(x, \phi)\)
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Probabilistic Quantification

Prediction

Model Inherits Exp. Error

Load variation imposed on prediction

Prediction with Uncertainty

Stochastic Stress Field

Targeted Random Sampling

Reliability Analysis

Prediction

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