Teaching a Verification and Validation Course using Simulations and Experiments with Paper Helicopter

Chanyoung Park
Joo-Ho Choi
Raphael T. Haftka
Motivation

- Why include experiments with paper helicopter?
  - Students have perspectives of both analysts and experimentalists
  - Paper helicopters can be easily built with paper and paper clips
  - Students can experience surprises in data from experiments

- Sharing experience of the paper helicopter project
  - Surprises in data collected by students
  - Lesson learned from the project
Paper Helicopter and Analysis

- Paper helicopter
  - Paper helicopters share the feature of autorotation with real helicopters
  - Autorotation of paper helicopters can be analyzed with a simple model

\[
F = mg - D \\
F = ma
\]
Drag Models

- **Quadratic model and Linear model**
  - Net force on a paper helicopter
    \[ F = mg - D \]
  - Quadratic drag model
    \[ D = \frac{1}{2} \rho_{air} A V^2 C_D \]
  - Linear drag model \((V_0 = 3\text{ft/sec})\)
    \[ D = \frac{1}{2} \rho_{air} C_D A V_0 V \]
  - Prediction for mass change
    (2 clips to 1 clip decreased by 18%)
Experiments for Model Validation

- Building three identical helicopters
  - Measuring mass
    1. Measuring mass of each helicopter with an accurate scale (i.e. 2.3±0.01 g)
    2. Measuring mass of all helicopters and use the average mass as a representative mass

- Collecting fall time data from experiments for three conditions
  - Fall time data of condition 1 for calibration
  - Fall time data of condition 2: change the number of clips (for validating drag models)
  - Fall time data of condition 3: change the height (for validating the constant $C_D$ assumption)
  - 10 fall time data for each condition
Calibration of drag coefficient

- A paper helicopter has variability in Drag coefficient
- We assume that CD of each helicopter follows the normal distribution
  \[ C_D \sim N(\mu_{C_d}, \sigma_{C_d}) \]
- Bayesian inference was suggested to reduce **epistemic uncertainty** due to finite data
Predictive Validation

- Prediction distribution of fall time

  - Predicting the fall time distribution for **different height** and **different mass** based on the calibrated drag coefficient distribution

- Considering the **epistemic uncertainty** in calibration is required when the prediction distribution is compared to data from experiments
Area Validation Metrics

- Comparing prediction distribution and data

  - Modeling the epistemic uncertainty in calibrated parameters in two ways
  - Area metric for p-box (2.5 and 97.5 percentiles)
  - Area metric for predictive distribution (Combining aleatory and epistemic uncertainties)
Example: Data of Student A

Fall time statistics

- COVs represent combination of variability in drag coefficient and uncertainty in test execution
- COVs are smaller than the other students
- The average time ratio of Helicopter 2 is very different from the other helicopters

<table>
<thead>
<tr>
<th>Condition</th>
<th>Helicopter 1</th>
<th>Helicopter 2</th>
<th>Helicopter 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight: 2 clips Height: 149 in (for calibration)</td>
<td>3.26 (3.6%)</td>
<td>2.91 (2.3%)</td>
<td>3.15 (3.1%)</td>
</tr>
<tr>
<td>Weight: 1 clip Height: 149 in</td>
<td>3.70 (1.9%)</td>
<td>3.84 (3.8%)</td>
<td>3.71 (1.6%)</td>
</tr>
<tr>
<td>Average time ratio</td>
<td>1.13</td>
<td>1.31</td>
<td>1.17</td>
</tr>
</tbody>
</table>
Example: Area metrics of Student A (Quadratic)

- **Area metric for p-box**

  - Area metric with p-box tries to capture the extreme discrepancy between the predicted variability and the observed variability

<table>
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<tr>
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<th>Helicopter2</th>
<th>Helicopter3</th>
</tr>
</thead>
<tbody>
<tr>
<td>149 in / 2 clips (calib)</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>149 in / 1 clips</td>
<td>0.017</td>
<td>0.411</td>
<td>0.026</td>
</tr>
</tbody>
</table>
Example: Area metrics of Student A (Linear)

Area metric for p-box

- The linear model represents the drag characteristic of Helicopter2
- It turned out that the quadratic model is for high Reynolds number ($>10^5$) and the linear model is for low Reynolds number
- Paper helicopter is in the intermediate regime

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<tr>
<td>149 in / 2 clips (calib)</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>149 in / 1 clips</td>
<td>0.513</td>
<td>0.008</td>
<td>0.360</td>
</tr>
</tbody>
</table>
Example: Data of Student B

**Fall time statistics**

- COVs represent combination of variability in drag coefficient and uncertainty in test execution
- COVs are larger than the other students
- The average time ratio of Helicopter 1 is very different from the other helicopters

<table>
<thead>
<tr>
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<th>Helicopter 2</th>
<th>Helicopter 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight: 2 clips Height: 148.5 in (for calibration)</td>
<td>3.78 (9.8%)</td>
<td>3.67 (9.0%)</td>
<td>3.56 (5.1%)</td>
</tr>
<tr>
<td>Weight: 1 clip Height: 148.5 in</td>
<td>4.16 (8.3%)</td>
<td>4.34 (3.4%)</td>
<td>4.29 (2.6%)</td>
</tr>
<tr>
<td>Average time ratio</td>
<td>1.10</td>
<td>1.19</td>
<td>1.20</td>
</tr>
</tbody>
</table>
Example: Area metrics of Student B (Quadratic)

Area metric for p-box

- Area metric with p-box tries to capture the extreme discrepancy between the predicted variability and the observed variability

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<tbody>
<tr>
<td>148.5 in / 2 clips (calib)</td>
<td>0.004</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>148.5 in / 1 clips</td>
<td>0.012</td>
<td>0.106</td>
<td>0.227</td>
</tr>
</tbody>
</table>

CDF of fall time

Area metric with p-box
Example: Area metrics of Student B (Linear)

- Area metric for p-box
  - The linear model is not as effective for Helicopter1 as the quadratic model

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</tr>
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<tbody>
<tr>
<td>148.5 in / 2 clips (calib)</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>148.5 in / 1 clips</td>
<td>0.116</td>
<td>0.048</td>
<td>0.013</td>
</tr>
</tbody>
</table>
Concluding Remarks

- Simulations and experiments with paper helicopter
  - Students had an opportunity to have perspectives of analysts and experimentalists
  - Students experienced difficulty in validation based on data from experiments using easily built and cheap paper helicopters

- Surprises in data from experiments
  - It was observed that one of helicopters has very different drag characteristic from the others that may be because helicopters operate in an intermediate Raynolds number regime
  - There was huge variability in drag characteristics of helicopters even if they were constructed by the same student very carefully
  - Students were exposed to this unexpected phenomenon
Acknowledgement

This work is supported by the U.S. Department of Energy, National Nuclear Security Administration, Advanced Simulation and Computing Program, as a Cooperative Agreement under the Predictive Science Academic Alliance Program (PSAAP), under Contract No. DE-NA0002378.
Thank you!
Drag Models

- **Quadratic model and Linear model**

  - Quadratic drag model is effective for high Raynolds number (>10^5) object
    
    \[
    d(t) = V_s t + \frac{\log(e^{-2V_s c t} + 1) - \log 2}{c} 
    \]
    
    \[c = \frac{\rho_{\text{air}} AC_D}{2m}\] **Quadratic model**

  - Linear drag model is effective for low Raynolds number (<1)
    
    \[
    d(t) = V_s \left( t + \frac{e^{-ct}}{c} \right) - \frac{V_s}{c} 
    \]
    
    \[c = \frac{\rho_{\text{air}} AC_D V_0}{2m}\] **Linear model**

  - The Raynolds number of paper helicopter is in intermediate regime