A-9100 INTRODUCTION

A-9110 SCOPE

(a) This Article contains a method for evaluating the adequacy of linear structural elements under combined loads, without determining principal stresses, by use of the stress ratio/interaction curve method. By using an interaction formula for combined stress states the ability of a linear structural element to withstand combined loads can then be determined provided the strength of the element under each individual load is known. The method can be applied to elastic and inelastic problems, including elastic and inelastic stability, and is useful when an exact stress analysis is not practical.

(b) A general interaction formula for three states of stress is given by the following:

\[
R_1^p + \left( R_2^q + R_3^r \right)^s \leq 1.0
\]  

(1)

where \( R_1 \), \( R_2 \), and \( R_3 \) are ratios of either individual stresses, stress resultants, or loads to their respective allowables; and the exponents \( p \), \( q \), \( r \), and \( s \) constitute the interaction relationship. These exponents are based upon experimental and/or theoretical considerations. Generally speaking, such an interaction is set up for each individual element in a structure (each beam, column, etc.), and each element will have its own set of exponents for the loads to which it is subjected.

(c) For elastic analysis of compact structures (those in which buckling need not be considered), interaction methods can be used to determine the yield surface. However, classical strength of material methods can also be used to obtain principal stresses, hence an interaction method is not of importance. For ultimate strength, an exact stress analysis is frequently impractical and interaction methods provide a useful alternative. In addition, for structures subject to more than one type of load which can cause instability (e.g., torsional and axial buckling of thin-walled tubes or pipes), interaction methods can again be used.

A-9120 NOMENCLATURE

(a) Definitions of the symbols used in this Article

\[ f_{uk} = \text{linearized ultimate bending stress for section factor} \ K \]
\[ f_{yk} = \text{linearized yield bending stress for section factor} \ K \]
\[ I = \text{moment of inertia} \]
\[ K = \text{section factor} \]
\[ M = \text{allowable bending moment} \]
\[ m = \text{applied moment} \]
\[ n = \text{interaction exponent} \]
\[ P = \text{axial load} \]
\[ Q_m = \text{first moment of the area between the neutral axis and outer fiber} \]
\[ R = \text{ratio of an individual stress or load to its allowable} \]
\[ S = \text{stress} \]
\[ S_o = \text{trapezoidal intercept stress} \]
\[ U = \text{stress field utilization factor} \]
\[ x = \text{centroidal axis,} \ x \ \text{direction} \]
\[ x' = \text{principal axis,} \ x' \ \text{direction} \]
\[ y = \text{centroidal axis,} \ y \ \text{direction} \]
\[ y_p = \text{distance from principal axis to intermediate fiber} \]
\[ y' = \text{principal axis,} \ y' \ \text{direction} \]
\[ \gamma = \text{plasticity factor} \]
\[ \phi = \text{angle between centroidal and principal axis, deg.} \]

(b) Indices used with the symbols in this Article

\( al = \text{allowable} \)
\( ap = \text{apparent} \)
\( b = \text{bending} \)
\( bc = \text{buckling} \)
\( c = \text{compression} \)
\( pl = \text{proportional limit} \)
\( s = \text{shear} \)
\( t = \text{tension} \)
\( to = \text{torsion} \)
\( u = \text{ultimate} \)
\( y = \text{yield} \)
\( 1, 2 = \text{locations across a section} \)
stress defined as follows:
(1) For elastic system analysis, the allowable stress $S_{a1}$ for components shall not exceed the lesser of $2.4S_m$ and $0.7S_u$ and the allowable stress $S_{a1}$ for linear supports shall not exceed the greater of $1.2S_y$ and $1.5S_m$, and shall not exceed $0.7S_u$.

(c) All structural shapes subject to buckling shall be governed by the requirements of NF-3300.

(f) Interaction equations which may be used for thin and thick-walled tubes and pipes, subject to various combinations of loads, are presented in Table A-9210(f)-1 (in the course of preparation).

(g) Interaction equations which may be used for flat, unperforated plates, subject to various combinations of loads, are presented in Table A-9210(g)-1 (in the course of preparation).

A-9300 ALLOWABLE LOADS AND STRESSES

A-9310 SCOPE

This subarticle provides criteria for determining the allowable loads for components or supports subject to the application of one or more loads. The allowable loads are to be based on the allowable stresses set forth in F-1331.2 or F-1341.5, as appropriate, for either elastic or plastic system analysis.
A-9320 Method

(a) The allowable load of a component or support under the application of a single load may be determined by experimental or analytical methods or both. The allowable loads of a component or support thus determined are to be modified by the effects discussed in A-9312 and A-9313 for use in the interaction equations of A-9200.

(b) An acceptable method of determining the allowable loads of beam shapes in pure bending or in bending in combination with direct loads and shear is the apparent stress method provided in A-9500.

(c) An acceptable method of determining the allowable loads of pipes and tubes in pure bending or in bending in combination with direct loads and shear is provided in A-9600 (in the course of preparation).

A-9400 New interaction equations

A-9410 Scope

Interaction equations other than those provided in A-9200 may be used for the analysis of components or supports, provided they are developed in accordance with the rules of this Section.

Table A-9210(d)-1

<table>
<thead>
<tr>
<th>Type of Load</th>
<th>Interaction Equation [Note (1)] and [Note (2)]</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple bending</td>
<td>$R_b &lt; 1$</td>
<td>$R_b = m/M$</td>
</tr>
<tr>
<td>Complex bending</td>
<td>$R_{bx'} + R_{by'} &lt; 1$</td>
<td>$R_{bx'} = m_{x'}/M_{x'}$, etc.</td>
</tr>
<tr>
<td>Simple shear</td>
<td>$R_s &lt; 1$</td>
<td>$R_s = S_{s}/S_{sal}$</td>
</tr>
<tr>
<td>Complex shear</td>
<td>$\sqrt{R_{sx}^2 + R_{sy}^2} &lt; 1$</td>
<td>$R_{sx'} = S_{sx'}/S_{sal}$, etc.; $S_{sx'}$ and $S_{sy'}$ are maximum shear stresses</td>
</tr>
<tr>
<td>Simple bending plus shear</td>
<td>$\sqrt{R_b^2 + R_s^2} &lt; 1$</td>
<td>[Note (3)]</td>
</tr>
<tr>
<td>Complex bending plus shear</td>
<td>$\sqrt{R_b^2 + R_s^2} &lt; 1$</td>
<td>$R_b' = R_{bx'} + R_{by'}$; [Note (1)]</td>
</tr>
<tr>
<td>Simple or complex bending plus tension</td>
<td>$R_s + R_t^n &lt; 1$</td>
<td>$R_t = P_t/P_{tal}$; to determine $n$ use A-9532</td>
</tr>
<tr>
<td>Simple or complex bending, tension, and shear</td>
<td>$\sqrt{(R_{by'} + R_{ny'})^2 + R_{sy}^2} &lt; 1$</td>
<td>[Note (3)]; to determine $n$ use A-9532; see A-9533</td>
</tr>
<tr>
<td>Simple or complex bending and compression</td>
<td>$R_s' + R_c &lt; 1$</td>
<td>[Note (3)] and [Note (4)]; $R_c = P_c/P_{cal}$</td>
</tr>
<tr>
<td>Simple or complex bending, compression, and shear</td>
<td>$\sqrt{(R_{by'} + R_{ny'})^2 + R_{sy}^2} &lt; 1$</td>
<td>[Note (3)] and [Note (4)]; see A-9535</td>
</tr>
</tbody>
</table>

NOTES:

(1) Allowable loads for use in interaction equations should be based on allowable stresses as defined in A-9300.

(2) All interaction ratios $R_i$ are positive by definition.

(3) As an alternate to the given interaction equation, the curve of Figure A-9210(d)-1 may be used.

(4) Amplification of bending moment by axial load shall be taken into account.

A-9500 Determination of Allowable Bending Strength of Beams by the Apparent Stress Method

A-9510 Scope

(a) This subarticle provides a method to calculate the strength of beams in the plastic range under pure bending or under bending combined with direct loads and shear. It is based on the work of Cozzzone$^5, 6$ and utilizes a fictitious stress called an apparent stress. This method shall not be used for the analysis of thin-walled tubes or pipes.