Appendix A-9000 Insertion of Interaction Method Limits
**ARTICLE A-9000**  
**INTERACTION METHOD**

**A-9100  INTRODUCTION**

**(a)** This Article contains a method for evaluating the adequacy of linear structural elements under combined loads, without determining principal stresses, by use of the stress ratio/interaction curve method. By using an interaction formula for combined stress states the ability of a linear structural element to withstand combined loads can then be determined provided the strength of the element under each individual load is known. The method can be applied to elastic and inelastic problems, including elastic and inelastic stability, and is useful when an exact stress analysis is not practical.

**(b)** A general interaction formula for three states of stress is given by the following:

\[
R_1^p + \left(R_2^q + R_3^r\right)^s \leq 1.0 \tag{1}
\]

where \(R_1, R_2,\) and \(R_3\) are ratios of either individual stresses, stress resultants, or loads to their respective allowables; and the exponents \(p, q, r,\) and \(s\) constitute the interaction relationship. These exponents are based upon experimental and/or theoretical considerations. Generally speaking, such an interaction is set up for each individual element in a structure (each beam, column, etc.), and each element will have its own set of exponents for the loads to which it is subjected.

**(c)** For elastic analysis of compact structures (those in which buckling need not be considered), interaction methods can be used to determine the yield surface. However, classical strength of material methods can also be used to obtain principal stresses, hence an interaction method is not of importance. For ultimate strength, an exact stress analysis is frequently impractical and interaction methods provide a useful alternative. In addition, for structures subject to more than one type of load which can cause instability (e.g., torsional and axial buckling of thin-walled tubes or pipes), interaction methods can again be used.

**A-9120  NOMENCLATURE**

**(a)** Definitions of the symbols used in this Article

- \(A = \) cross-sectional area
- \(c = \) distance from neutral axis to outermost fiber
- \(e = \) strain
- \(f_{ap} = \) linearized allowable bending stress (apparent stress)
- \(f_{uk} = \) linearized ultimate bending stress for section factor \(K\)
- \(f_{yk} = \) linearized yield bending stress for section factor \(K\)
- \(I = \) moment of inertia
- \(K = \) section factor
- \(M = \) allowable bending moment
- \(m = \) applied moment
- \(n = \) interaction exponent
- \(P = \) axial load
- \(Q_m = \) first moment of the area between the neutral axis and outer fiber
- \(S_o = \) trapezoidal intercept stress
- \(U = \) stress field utilization factor
- \(x = \) centroidal axis, \(x\) direction
- \(x' = \) principal axis, \(x'\) direction
- \(y = \) centroidal axis, \(y\) direction
- \(y' = \) principal axis, \(y'\) direction
- \(\gamma = \) plasticity factor
- \(\phi = \) angle between centroidal and principal axis, deg.

**(b)** Indices used with the symbols in this Article

- \(al = \) allowable
- \(ap = \) apparent
- \(b = \) bending
- \(bc = \) buckling
- \(c = \) compression
- \(pl = \) proportional limit
- \(s = \) shear
- \(t = \) tension
- \(to = \) torsion
- \(u = \) ultimate
- \(y = \) yield

1, 2 = locations across a section

**A-9200  INTERACTION EQUATIONS**

**A-9210  SCOPE**

**(a)** This subarticle provides interaction equations based on experimental data for a number of common structural shapes.
(b) Allowable loads and stresses for the interaction equations presented herein shall be determined in accordance with A-9300.

(c) Interaction equations for combinations of loads other than those specified herein may be used, provided they are developed in accordance with the rules of A-9400.

(d) Interaction equations, which may be used for common beam shapes subject to various combinations of loads, are presented in Table A-9210(d)-1. As an alternative to some of the interaction equations given in Table A-9210(d)-1, the curve in Figure A-9210(d)-1 may be used.

(e) All structural shapes subject to buckling shall be governed by the requirements of NF-3300.

(f) Interaction equations which may be used for thin- and thick-walled tubes and pipes, subject to various combinations of loads, are presented in Table A-9210(f)-1 (in the course of preparation).

(g) Interaction equations which may be used for flat, unperforated plates, subject to various combinations of loads, are presented in Table A-9210(g)-1 (in the course of preparation).

A-9300 ALLOWABLE LOADS AND STRESSES

A-9310 SCOPE

This subarticle provides criteria for determining the allowable loads for components or supports subject to the application of one or more loads. The allowable loads are to be based on the allowable stresses set forth in F-1331.2 or F-1341.5, as appropriate, for either elastic or plastic system analysis.

A-9311 Material Properties

The material properties used in developing the allowable component or support loads or stresses shall be based on Section II, Part D, Subpart 1, and included in the Design Report.

A-9312 Strain Rate Effects

Strain rate effects on material properties may be considered if justified in the Design Report.

A-9313 Temperature Effects

Temperature effects on the allowable component or support loads or stresses shall be considered and justified in the Design Report.

A-9314 Allowable Load

The allowable load of a component or support is defined as the lesser of (a) through (d).

(a) The load at which the most severely stressed fiber reaches the allowable stress defined in F-1331.2 or F-1341.5, as appropriate.

(b) The load at which either strain or deformation exceeds the limits provided by the component or support Design Specification.

(c) The load at which loss of component or support function occurs, as defined by the component or support Design Specification.

(d) The allowable buckling load as defined in F-1334.3.

Mandatory Appendix XXVII-3400 for components or Level D critical buckling rules in NF-3300 for supports.

Figure A-9210(d)-1

Interaction Curve for Beams Subject to Bending and Shear or to Bending, Shear, and Direct Loads

A-9313 Temperature Effects

Temperature effects on the allowable component or support loads or stresses shall be considered and justified in the Design Report.

A-9314 Allowable Load

The allowable load of a component or support is defined as the lesser of (a) through (d).

(a) The load at which the most severely stressed fiber reaches the allowable stress defined in F-1331.2 or F-1341.5, as appropriate.

(b) The load at which either strain or deformation exceeds the limits provided by the component or support Design Specification.

(c) The load at which loss of component or support function occurs, as defined by the component or support Design Specification.

(d) The allowable buckling load as defined in F-1334.3.
A-9320 METHOD

(a) The allowable load of a component or support under the application of a single load may be determined by experimental or analytical methods or both. The allowable loads of a component or support thus determined are to be modified by the effects discussed in A-9312 and A-9313 for use in the interaction equations of A-9200.

(b) An acceptable method of determining the allowable loads of beam shapes in pure bending or in bending in combination with direct loads and shear is the apparent stress method provided in A-9500.

(c) An acceptable method of determining the allowable loads of pipes and tubes in pure bending or in bending in combination with direct loads and shear is provided in A-9600 (in the course of preparation).

A-9400 NEW INTERACTION EQUATIONS

A-9410 SCOPE

Interaction equations other than those provided in A-9200 may be used for the analysis of components or supports, provided they are developed in accordance with the rules of this Section.

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Table A-9210(d)-1

Interaction Equations for Common Beam Shapes

<table>
<thead>
<tr>
<th>Type of Load</th>
<th>Interaction Equation [Note (1)] and [Note (2)]</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple bending</td>
<td>$R_b &lt; 1$</td>
<td>$R_b = m/M$</td>
</tr>
<tr>
<td>Complex bending</td>
<td>$R_{bx} + R_{by} &lt; 1$</td>
<td>$R_{bx} = m_{x}/M_{x}$, etc.</td>
</tr>
<tr>
<td>Simple shear</td>
<td>$R_s &lt; 1$</td>
<td>$R_s = S_{s}/S_{sal}$</td>
</tr>
<tr>
<td>Complex shear</td>
<td>$\sqrt{R_{sx}^2 + R_{sy}^2} &lt; 1$</td>
<td>$R_{sx} = S_{sx}/S_{sxal}$, $S_{sx}$ and $S_{sy}$ are maximum shear stresses</td>
</tr>
<tr>
<td>Simple bending plus shear</td>
<td>$\sqrt{R_b^2 + R_s^2} &lt; 1$</td>
<td>[Note (3)]</td>
</tr>
<tr>
<td>Complex bending plus shear</td>
<td>$\sqrt{R_b^2 + R_s^2} &lt; 1$</td>
<td>$R_b = R_{bx} + R_{by}$; $R_s = \sqrt{R_{sx}^2 + R_{sy}^2}$; [Note (1)]</td>
</tr>
<tr>
<td>Simple or complex bending plus tension</td>
<td>$R_b + R_c &lt; 1$</td>
<td>$R_c = P_c/P_{cal}$; to determine $n$ use A-9532</td>
</tr>
<tr>
<td>Simple or complex bending, tension, and shear</td>
<td>$\sqrt{(R_{by}^2 + R_{ty}^2) + R_s^2} &lt; 1$</td>
<td>[Note (3)]; to determine $n$ use A-9532; see A-9533</td>
</tr>
<tr>
<td>Simple or complex bending and compression</td>
<td>$R_b + R_c &lt; 1$</td>
<td>[Note (3)] and [Note (4)]; $R_c = P_c/P_{cal}$</td>
</tr>
<tr>
<td>Simple or complex bending, compression, and shear</td>
<td>$\sqrt{(R_{by}^2 + R_{ty}^2) + R_s^2} &lt; 1$</td>
<td>[Note (3)] and [Note (4)]; see A-9535</td>
</tr>
</tbody>
</table>

NOTES:
(1) Allowable loads for use in interaction equations should be based on allowable stresses as defined in A-9300.
(2) All interaction ratios $R_i$ are positive by definition.
(3) As an alternate to the given interaction equation, the curve of Figure A-9210(d)-1 may be used.
(4) Amplification of bending moment by axial load shall be taken into account.

A-9420 METHOD

Any new interaction equations to be used shall be included and justified in the Design Report. They may be justified by one of the following:

(a) common appearance in appropriate technical literature or in industry codes or standards;

(b) experimental development which includes a variation of all types of loads or stresses that appear in the interaction equations. Such a load or stress variance shall bound the loads or stresses to which the component is subjected;

(c) theoretical development which includes testing to verify the interaction equations developed.

A-9500 DETERMINATION OF ALLOWABLE BENDING STRENGTH OF BEAMS BY THE APPARENT STRESS METHOD

A-9510 SCOPE

(a) This subarticle provides a method to calculate the strength of beams in the plastic range under pure bending or under bending combined with direct loads and shear. It is based on the work of Cozzzone\textsuperscript{5, 6} and utilizes a fictitious stress called an apparent stress. This method shall not be used for the analysis of thin-walled tubes or pipes.
of:

(-a) for elastic component analysis, the allowable stress \( S_{a1} \) shall not exceed the lesser of \( 2.4 S_m \) or \( 0.7 S_u \).

(-b) for linear type supports, the allowable stress \( S_{a1} \) shall not exceed the greater of \( 1.2 S_y \) and \( 1.5 S_m \), but not larger than \( 0.7 S_u \).

\[ \text{distribution corresponds with the stress–strain relationship for the material. An approximation of this distribution has been obtained, which enables the prediction of the effects of shape and material properties on bending in the plastic range. This method has the advantage that strain hardening may be taken into account.} \]

\[ (c) \text{ The methods provided herein may also be used for the analysis of beams with cutouts or notches, provided that the geometric properties are based on the net area at the cutout or notch.} \]

\[ (d) \text{ The effect of cyclic loading should be evaluated independently, where appropriate.} \]

**A-9520 SIMPLE BENDING**

**A-9521 Simple Bending — Symmetrical Sections**

(a) The method given below may be used when the resultant applied moment vector is parallel to a principal axis which is also an axis of symmetry.

\[ \text{Table A-9521(b)-1} \]

<table>
<thead>
<tr>
<th>Flanges Only</th>
<th>Solid Round</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K = 1 )</td>
<td>( K = 1 ) to ( 1.5 )</td>
</tr>
</tbody>
</table>

\[ \text{Table A-9521(b)-1} \]

(1) Using the method outlined in A-9540, derive the relationship between the allowable stress \( S_{a1} \) and the linearized allowable stress \( f_{ap} \) for the proper section factor \( K \). An example calculation for SA-672 A50 material at 600°F is provided in A-9542, with the resulting relationship shown in Figure A-9542-1.
Enter this strain $e_1$ on the stress–strain curve and obtain the corresponding stress from the stress–strain curve. Use this stress value as the allowable stress $S_{ai}$ and, with the K value for the part with the smaller $c$, use the method of A-9521(c) to obtain $f_{ap}$ for this part.

(4) Multiply the $f_{ap}$ value for each part by $l/c$ of each part and add the two to obtain the total allowable moment $M$.

A-9523 Complex Bending — Symmetrical and Unsymmetrical Sections

This condition occurs when the resultant applied moment vector is not parallel to a principal axis.

A-9523.1 Sign Convention and Nomenclature. In Figure A-9523.1-1, let $x$ and $y$ represent two mutually perpendicular centroidal axes, and let $x'$ and $y'$ represent the principal axes.

A-9523.2 Resolution of Complex Bending Into Simple Bending. Any case of complex bending may be resolved into two cases of simple bending about the principal axes of the section. The principal axes are defined as mutually perpendicular centroidal axes about which the moments of inertia are a maximum and minimum, respectively, and about which the product moment of inertia is zero. The procedure is given in (a) through (d) below.

(a) Determine the principal axes $x'$ and $y'$. If they cannot be determined by inspection, obtain $I_x$, $I_y$, and $I_{xy}$ about any arbitrary pair of centroidal axes.

$$\tan 2\phi = 2I_{xy}/(I_y - I_x)$$

(b) Using the $x'$ axis as a reference, determine the allowable moment $M_{x'}$ as described under simple bending (A-9521 and A-9522).

(c) Using the $y'$ axis as a reference, determine the allowable moment $M_{y'}$ as described under simple bending (A-9521 and A-9522).

(d) For use in the interaction equations of A-9210, moments in the global axis shall be resolved into moments about the principal axes by use of the following relationships:

$$m_{x'} = m_x \cos \phi + m_y \sin \phi$$

$$m_{y'} = m_x \sin \phi + m_y \cos \phi$$

A-9530 BENDING COMBINED WITH A STRESS FIELD

A-9531 Interaction — Simple or Complex Bending and Shear

The maximum shear stress in a beam usually occurs at the principal (neutral) axis where the bending stress is zero. The maximum bending stress occurs at an extreme fiber where the shear stress is usually zero.

(a) In the elastic range, the distribution of shear and bending stresses (see Figure A-9531-1) is usually such that the most critical point in the section is at either the principal axis or the extreme fiber. This is true on a rectangular section since the shear distribution across the section is parabolic and the bending distribution is linear. If the shear distribution has been elliptical, every point in the cross section will be equally critical in combined stress based on circular interaction.

(b) In the plastic range, however, the distribution of the shear stress as well as the bending stress differs from that in the elastic range. This results in intermediate points which frequently become more critical in combined stress than either the shear stress at the principal axis or the bending stress at the extreme fiber.

(c) To find the most critical point would require the calculation of combined stresses at a series of points across the section. This procedure would not only be laborious but probably incorrect in the conservative direction, since there would undoubtedly be some redistribution of stress.
away from the most critical point, although the exact nature of this redistribution appears to be extremely difficult to determine. Therefore, the procedure of (1) through (4) below shall be used.

(1) The method used to determine the shear flows for simple or complex bending shall be included and justified in the Design Report.

(2) For complex bending, the maximum principal shear stresses \( S_{x'x'} \) and \( S_{y'y'} \) shall be determined for use in the interaction equations of A-9210.

(3) For simple or complex bending, the allowable shear stress \( S_{sax} \) shall be taken as 0.6\( S_{sax}' \).

(4) The allowable moments shall be determined as in A-9521 or A-9522, as appropriate.

**A-9532 Interaction — Simple or Complex Bending and Tension**

(a) The allowable moments for simple or complex bending shall be determined from A-9520, as appropriate.

(b) The allowable tensile load \( P_{tl} \) shall be taken as \( A S_{atl} \).

(c) The interaction exponent \( n \) for use in the interaction equations of A-9210(d) shall be determined from (1) through (3) below.

(1) Determine \( Ac/2Qm \) for use in obtaining the interaction exponent. If the section is unsymmetrical, take \( c \) for the side for which axial and bending stresses are of opposite sign. For complex bending, obtain both \( Ac/2c_{x'} \) and \( Ac/2c_{y'} \). In obtaining \( Ac/2c_{x'} \) if the section is unsymmetrical about the \( x' \) axis, take \( c_{x'} \) for the side for which axial stress and stress due to \( m_{x} \) are of opposite sign. Obtain \( Ac/2c_{y'} \), in the same manner.

(2) The material plasticity factor for use in obtaining the interaction exponent is \( \gamma = 0.90 \) for all materials.

(3) Using \( Ac/2Qm \) determined above in (1) and \( \gamma \) determined above in (2), obtain \( n \) from Figure A-9532(c)(3)-1. To determine the interaction exponent for complex bending, obtain both \( n_{x'} \) and \( n_{y'} \) from Figure A-9532(c)(3)-1 and determine a combined \( n \) using \( n = (n_{x} + n_{y}) / R_{b} \).

**A-9533 Interaction — Simple or Complex Bending, Tension, and Shear**

When bending acts in addition to tension and shear, allowables shall be determined as provided in (a) through (c) below.

(a) Follow the procedure outlined in A-9531 and A-9532 to obtain \( S_{sax}, P_{tl}, M_{x'}, M_{y'}, \) and \( n \).

(b) Plot the curve of \( R_{b} + R_{t} \) and the intersection of \( R_{b} \) and \( R_{t} \). Call this point A. Obtain \( a = 0A \) and \( b = 0B \).

(c) Obtain the bending-tension utilization factor for use in Figure A-9210(d)-1:

\[ U_{bt} = a/b \]

**A-9534 Interaction — Simple or Complex Bending and Compression**

(a) When compression acts in addition to simple or complex bending, the applied moment \( m \) shall take into account the additional bending caused by the compressive load.

(b) The allowable moments for simple or complex bending shall be determined from A-9520, as appropriate.

(c) The allowable compressive load \( P_{cal} \) shall be taken as the lesser of \( AS_{aj} \) and the allowable buckling load \( P_{cal} \) in either Mandatory Appendix XXVII-3400 for components or NF-3300 for supports.

**A-9535 Interaction — Simple or Complex Bending and Shear**

When bending acts in addition to tension and shear, the following procedure shall be used to determine the interaction relationships for use in A-9210.

(a) Follow the procedure in A-9534 to obtain the applied and allowable moments and the allowable compressive load.

(b) Obtain the bending-compression utilization factor \( U_{bc} \).

(c) \( U_{bc} \) may be used in Figure A-9210(d)-1 by replacement of \( U_{bt} \) by \( U_{bc} \).

**A-9540 PROCEDURE FOR DETERMINATION OF ALLOWABLE BENDING STRESS**

**A-9541 Derivation of Linearized Allowable Bending Stress for Any Material**

A fictitious bending stress, called the linearized allowable bending stress \( f_{ap} \), may be used for establishing the bending strength of a material. This method assumes that in the plastic region the nonlinear stress–strain relationship for a particular section and material can be approximated by a trapezoidal shape as shown in Figure A-9541-1.

The stress \( S_{o} \) is a fictitious stress which is assumed to exist at the neutral axis or at zero strain. The value of \( S_{o} \) is determined by requiring that the internal moment of the engineering stress–strain curve must equal the internal moment of the assumed trapezoidal shape. Thus, the total moment capacity of a symmetrical section may be expressed as follows:

\[
M = 2S_{o} \int yda + (S_{al} - S_{o})(1/c) \\
= 2S_{o}Qm + (S_{al} - S_{o})(1/c) \\
= (1/c)[S_{al} + S_{o}(2Qm/(1/c) - 1)] \\
= (1/c)[S_{al} + S_{o}(K - 1)]
\]

Thus, the linearized allowable stress for any section factor \( K \) becomes
and the moment capacity of the section is as follows:

\[ M = f_{apk}/c \]  \hspace{1cm} (4)

An alternate method of determining \( f_{ap} \) for any section factor is to express it in terms of the linearized allowable bending stress for a 1.5 stress factor:

\[ f_{apk} = f_{ap 1.5} + (K - 1.5)S_0 \]  \hspace{1cm} (5)

When the allowable stress \( S_{al} \) is equal to the ultimate stress \( S_u \), eqs. (3) and (5) become

\[ f_{uk} = S_u + (K - 1)S_{ou} \]  \hspace{1cm} (6)

\[ f_{uk} = f_{u1.5} + (K - 1.5)S_{ou} \]  \hspace{1cm} (7)

When the allowable stress \( S_{al} \) is equal to the yield stress \( S_y \), eqs. (3) and (5) become

\[ f_{yk} = S_y + (K - 1)S_{oy} \]  \hspace{1cm} (8)

\[ f_{yk} = f_{y1.5} + (K - 1.5)S_{oy} \]  \hspace{1cm} (9)

where \( S_{ou} \) and \( S_{oy} \) are the trapezoidal intercept stresses corresponding to \( S_u \) and \( S_y \).

In order to calculate the allowable moment of a given beam cross section by the use of eqs. (3) and (4), the intercept stress \( S_o \) must first be determined using the allowable moment, determined either by test or an exact stress analysis, that corresponds to a known value of \( K \).

Such effort would negate the advantage of this method. On the other hand, the values of \( S_{ou} \) and \( S_{oy} \) have been calculated for about 50 materials\(^7\) and are shown in Figure A-9541-2. These curves may be utilized for the carbon, low, and high alloy steels given in Section II, Part D, Subpart 1. The corresponding values of \( f_{u1.5} \) and \( f_{y1.5} \) are shown in Figure A-9541-3.

Once \( f_{uk} \) and \( f_{yk} \) are determined from Figure A-9541-3 and eqs. (6) through (9), \( f_{ap} \) for any value of the allowable stress \( S_{al} \) may be determined as shown in (a) through (c) below.

- (a) For \( S_{al} \leq S_{pl} \), where \( S_{pl} \) is the proportional limit stress (Figure A-9541-4),

\[ f_{apk} = S_{al} \]  \hspace{1cm} (10)

- (b) For \( S_{pl} \leq S_{al} \leq S_y \),

\[ f_{apk} = S_{pl} + Z_{1k}(S_{al} - S_{pl}) \]  \hspace{1cm} (11)

where

\[ Z_{1k} = \left( f_{yk} - S_{pl} \right) / \left( S_y - S_{pl} \right) \]  \hspace{1cm} (12)

- (c) For \( S_y < S_{al} \leq S_u \),

\[ f_{apk} = f_{yk} + Z_{2k}(S_{al} - S_y) \]  \hspace{1cm} (13)

where

\[ Z_{2k} = \left( f_{uk} - f_{yk} \right) / \left( S_u - S_y \right) \]  \hspace{1cm} (14)
Figure A-9533(b)-1
Interaction Curve for Bending and Tension

Figure A-9541-1
Trapezoidal Stress–Strain Relationship

Assumed trapezoidal shape

Engineering stress-strain curve
Figure A-9541-2
Ultimate and Yield Trapezoidal Intercept Stresses

Figure A-9541-3
Linearized Ultimate and Yield Bending Stresses for Rectangular Section
Figure A-9541-4
Proportional Limit as a Function of Yield Stress

Proportional Limit $S_{pl}$, ksi

Yield Stress $S_Y$, ksi
Example Illustrating the Derivation of Linearized Allowable Bending Stress for SA-672 A50 Material at 600°F

The values of $S_y$ and $S_u$ are given:

$S_u = 50.0$ ksi ultimate tensile strength at 600°F (Section II, Part D, Subpart 1, Table U)

$S_y = 20.0$ ksi yield strength at 600°F (Section II, Part D, Subpart 1, Table Y-1)

The following values of $S_{oy}$ and $S_{ou}$ are found from Figure A-9541-2:

$S_{oy} = 11$ ksi

$S_{ou} = 44$ ksi

Using eqs. A-9541(6) and A-9541(8),

\[ f_{u1.5} = 50 + 0.5(44) = 72 \text{ ksi} \]

\[ f_{y1.5} = 20.0 + 0.5(11) = 25.5 \text{ ksi} \]

or, using Figure A-9541-3,

\[ f_{u1.5} = 72 \text{ ksi} \]

\[ f_{y1.5} = 25 \text{ ksi} \]

Using eqs. A-9541(7) and A-9541(9), the value of $f_{uk}$ and $f_{yk}$, for any $K$, is determined. For example, if $K = 1.9, 1.7, 1.3,$ and $1.1$, then

\[ f_{u1.9} = 72.0 + 0.4(44.0) = 89.6 \text{ ksi} \]

\[ f_{u1.7} = 72.0 + 0.2(44.0) = 80.8 \text{ ksi} \]

\[ f_{u1.3} = 72.0 + (-0.2)(44.0) = 63.2 \text{ ksi} \]

\[ f_{u1.1} = 72.0 + (-0.4)(44.0) = 54.4 \text{ ksi} \]

\[ f_{y1.9} = 25.5 + 0.4(11.0) = 29.8 \text{ ksi} \]

\[ f_{y1.7} = 25.5 + 0.2(11.0) = 27.6 \text{ ksi} \]

\[ f_{y1.3} = 25.5 + (-0.2)(11.0) = 23.2 \text{ ksi} \]

\[ f_{y1.1} = 25.5 + (-0.4)(11.0) = 21.0 \text{ ksi} \]

From Figure A-9541-4, $S_{p1} = 13.0$ ksi. Using the above data, Figure A-9542-1 may be obtained for $K = 1.9, 1.5,$ and $1.1$.

From F.1334.1, the allowable stress is the lesser of $1.2S_y$ and $0.7S_u$:

\[ 1.2S_y = 1.2(20) = 24 \text{ ksi} \]

\[ 0.7S_u = 0.7(50) = 35.0 \text{ ksi} \]

Therefore, $S_{al} = 24.0$ ksi and

\[ S_y = 20 \text{ ksi} \]

\[ S_u = 50 \text{ ksi} \]

\[ S_{al} < S_{al} < S_u \]

From eq. A-9541(c)(14), with $K = 1.5$,

\[ Z = \frac{(72 - 25.5)}{(50 - 20)} = 1.55 \]

From eq. A-9541(c)(13),

\[ f_{ap} = 25.5 + 1.55(24 - 20) \]

\[ = 31.7 \text{ ksi} \]

Alternatively, from Figure A-9542-1, $f_{ap} = 31.5$ ksi.
Figure A-9542-1
Linearized Bending Stress Versus Allowable Stress for SA-672 A50 Material at 600°F (316°C)
Division 3, Subsection WD cross reference cleanup
WD-3200  DESIGN RULES FOR PLATE- AND SHELL-TYPE INTERNAL SUPPORT STRUCTURES

WD-3210  GENERAL REQUIREMENTS

Plate- and shell-type internal support structures are fabricated from plate and shell elements and are normally subjected to a biaxial stress field. The design procedures shall ensure that deformation criteria are satisfied as required in WD-1110(b). The design procedures recognized for the plate and shell elements are:

(a) Design by analysis based on maximum shear stress theory using elastic analyses with the rules of WD-3220 (for Design, Level A and C);

(b) The theory of failure used in the rules of this subarticle shall be the maximum shear stress theory using elastic analyses with the rules of WD-3220 (for Design, Level A and C);

(c) The critical buckling stress shall be calculated; see WD-3229.

(d) Protection against nonductile fracture shall be provided. An acceptable procedure for nonductile failure prevention is given in Section III Appendices, Nonmandatory Appendix G.

WD-3212  Basis for Determining Stresses

The theory of failure used in the rules of this subarticle for combining stresses is the maximum shear stress theory. The maximum shear stress at a point is equal to one-half the difference between the algebraically largest and the algebraically smallest of the three principal stresses at the point.

WD-3213  Terms Relating to Stress Analysis

Terms used in this Subsection relating to stress analysis are defined in the following subparagraphs. The following stress terms discuss a variety of applied loads in order to achieve understanding and clarity but not all of these loads are applicable to internal support structures.

WD-3213.1  Stress Intensity. Stress intensity is defined as twice the maximum shear stress, which is the difference between the algebraically largest principal stress and the algebraically smallest principal stress at a given point. Tensile stresses are considered positive and compressive stresses are considered negative.

WD-3213.2  Normal Stress. Normal stress is the component of stress normal to the plane of reference. This is also referred to as direct stress. Usually the distribution of normal stress is not uniform through the thickness of a plate, so this stress is divided into two components, one uniformly distributed and equal to the average stress across the thickness of the section under consideration and the other varying from this average value across the thickness.

WD-3213.3  Shear Stress. Shear stress is the component of stress tangent to the plane of reference.

WD-3213.4  Membrane Stress. Membrane stress is the component of normal stress that is uniformly distributed and equal to the average stress across the thickness of the section under consideration.

WD-3213.5  Bending Stress. Bending stress is the component of normal stress that varies across the thickness. The variation may or may not be linear.

WD-3213.6  Primary Stress. Primary stress is any normal stress or shear stress developed by an imposed loading that is necessary to satisfy the laws of equilibrium of external and internal forces and moments. The basic characteristic of a primary stress is that it is not self-limiting. Primary stresses that considerably exceed the yield strength will result in failure or, at least, in gross distortion. Primary membrane stress is divided into general and local categories. A general primary membrane stress is one that is so distributed in the structure that no redistribution of load occurs as a result of yielding. Examples of primary stress are

(a) general membrane stress in a circular cylindrical shell or a spherical shell due to pressure, inertial, or distributed loads

(b) bending stress in the central portion of a flat head due to pressure or inertial loads.

Refer to Table WD-3217-1 for examples of primary stress.

WD-3213.7  Secondary Stress. Secondary stress is a normal stress or a shear stress developed by the constraint of adjacent material or by self-constraint of the structure. The basic characteristic of a secondary stress is that it is self-limiting. Local yielding and minor distortions can satisfy the conditions that cause the stress to occur and failure from one application of the stress is not to be expected. Examples of secondary stress are

(a) general thermal stress [WD-3213.12(a)];

(b) bending stress at a gross structural discontinuity.

Refer to Table WD-3217-1 for examples of secondary stress.

WD-3213.8  Local Primary Membrane Stress. Cases arise in which a membrane stress produced by pressure or other mechanical loading and associated with a
to determination of the stress intensity values. The allowable value of the stress intensity is \( S_m \) at the Design Temperature.

**WD-3221.2 Local Primary Membrane Stress Intensity.** This stress intensity (derived from \( P_L \) in Figure WD-3221-1) is derived from the average value across the thickness of a section of the local primary stresses \( \) produced by Design Mechanical Loads, but excluding all secondary and peak stresses. Averaging is to be applied to the stress components prior to determination of the stress intensity values. The allowable value of the stress intensity is 1.5 \( S_m \).

**WD-3221.3 Primary Membrane (General or Local) Plus Primary Bending Stress Intensity.** This stress intensity \( P_m \) [derived from \( P_m \) or \( P \) in Figure WD-3221-1] is derived from the highest value across the thickness of a section of the general or local primary membrane stresses plus primary bending stresses produced by specified Design Mechanical Loads, but excluding all secondary and peak stresses. For solid rectangular sections, the allowable value of this stress intensity is \( 1.5S_m \). For other than solid rectangular sections, a value of \( \alpha \) times the limit established in WD-3221.1 may be used, where the factor \( \alpha \) is defined as the ratio of the load set producing a fully plastic section to the load set producing initial yielding in the extreme fibers of the section. In the evaluation of the initial yield and fully plastic section capacities, the ratios of each individual load in the respective load set to each other load in that load set shall be the same as the respective ratios of the individual loads in the specified design load set. The value of \( \alpha \) shall not exceed 1.5. The propensity for buckling of the part of the section that is in compression shall be investigated. The \( \alpha \) factor is not permitted for Level D Service Limits when inelastic component analysis is used as permitted in Section III Appendices, Nonmandatory Appendix F, F-1340.

**WD-3222 Level A Service Limits**

The Level A Service Limits shall be satisfied for the normal loadings for which these limits are designated in the Design Specification and are the limits of this paragraph and WD-3227. The design stress intensity values \( S_m \) at the coincident normal operating temperature are described in WD-3112.3. The limits are summarized by Figure WD-3222-1.

**WD-3222.1 General Primary Membrane Stress Intensity.** This stress intensity (derived from \( P_m \) in Figure WD-3222-1) is derived from the average value across the thickness of a section of the general primary stresses \( \) produced by the specified mechanical loads.
Step 2. For each type of stress cycle, determine the alternating stress intensity $S_{alt}$ by the procedures of WD-3216.1 or WD-3216.2 above. Call these quantities $S_{alt1}$, $S_{alt2}$, $S_{alt3}$, ..., $S_{alt n}$.

Step 3. For each value $S_{alt1}$, $S_{alt2}$, $S_{alt3}$, ..., $S_{alt n}$, use the applicable design fatigue curve (Section III Appendices, Mandatory Appendix I) to determine the maximum number of repetitions that would be allowable if this type of cycle were the only one acting. Call these values $N_1$, $N_2$, $N_3$, ..., $N_n$.

Step 4. For each type of stress cycle, calculate the usage factors $U_1$, $U_2$, $U_3$, ..., $U_n$, from $U_1 = n_1/N_1$, $U_2 = n_2/N_2$, $U_3 = n_3/N_3$, ..., $U_n = n_n/N_n$.

Step 5. Calculate the cumulative usage factor $U$ from $U = U_1 + U_2 + U_3 + ... + U_n$.

Step 6. The cumulative usage factor $U$ shall not exceed 1.0.

WD-3222.6 Stress Ratcheting. It should be noted that under certain combinations of steady-state and cyclic loadings, there is a possibility of large distortions developing as the result of ratchet action; that is, the deformation increases by a nearly equal amount for each cycle. Ratcheting can occur in a structure that is subjected to tension increases by a nearly equal amount for each cycle. That deformation may occur as the result of ratchet action; that is, the deformation increases by a nearly equal amount for each cycle.

(a) The limiting value of the maximum cyclic thermal stress permitted in a portion of an axisymmetric shell or plate with steady-state loading in order to prevent cyclic progressive distortion is determined as follows. Let

$$y' = \text{maximum allowable range of thermal stress computed on an elastic basis divided by the yield strength } S_y \text{ taken at the average temperature of the transient under consideration}$$

$$x = \text{maximum general membrane stress divided by the yield strength } S_y \text{ taken at the average temperature of the transient under consideration}$$

NOTE: For both $x$ and $y'$, it is permissible to use $1.5S_m$ whenever it is greater than $S_y$.

(1) Case 1: linear variation of temperature through the wall

$$y' = 1/x \text{ for } 0 < x < 0.5$$

$$y' = 4(1 - x) \text{ for } 0.5 < x < 1.0$$

(2) Case 2: parabolic, constantly increasing or constantly decreasing, variation of temperature through the wall

$$y' = 5.2(1 - x) \text{ for } 0.615 \leq x < 1.0 \text{ and, approximately, for } x < 0.615 \text{ as follows:}$$

$$y' = 4.65, 3.55, \text{ and } 2.70, \text{ for } x = 0.3, 0.4, \text{ and } 0.5, \text{ respectively.}$$

(b) Use of yield strength $S_y$ in the above relations instead of the proportional limit allows a small amount of growth during each cycle until strain hardening raises the proportional limit to $S_y$. If the yield strength of the material is higher than two times the $S_y$ value for the maximum number of cycles on the applicable fatigue curve of Section III Appendices, Mandatory Appendix I for the material, the latter value shall be used if there is to be a large number of cycles because strain softening may occur.

(c) Similar methodology shall be used for mechanically-applied ratcheting loads specified in the Design Specification. The Design Specification may prescribe specific methodology.

WD-3222.7 Deformation Limits. Any deformation limits prescribed by the Design Specification shall be satisfied. See WD-3125.

WD-3224 Level C Service Limits

The Level C Service Limits shall be satisfied for off-normal loadings for which they are designated by the Design Specification and are summarized by Figure WD-3224.1. Dynamic instability shall be considered in meeting the load, stress, and deformation limits.

WD-3224.1 Primary Stress Intensity Limits. The permissible values for Level C Service Limits shall be taken as 150% of the values given in WD-3221. See WD-3221.3 when evaluating other than solid rectangular sections.

WD-3224.2 Special Stress Limits. The permissible values for special stress limits shall be taken as 150% of the values given in WD-3227. The requirements of WD-3224.2(c) and WD-3227.3 need not be satisfied.

WD-3224.3 Deformation Limits. Any deformation limits prescribed by the Design Specification shall be satisfied. See WD-3125.

WD-3225 Level D Service Limits

If the Design Specification specifies any accident loadings for which Level D Service Limits are designated [WA-2123.4(b)], the rules contained in Section III Appendices, Nonmandatory Appendix F shall be used in evaluating these loadings, independently of all other operating loadings. The rules in Section III Appendices, Nonmandatory Appendix F, Table F-1200-1 for plate- and shell-type supports shall apply. The requirements of WD-3229.3 shall be satisfied. Any deformation limits prescribed by the Design Specification shall be satisfied. See WD-3125.

WD-3227 Special Stress Limits

The following deviations from the basic stress limits are provided to cover special operating loadings or configurations. Some of these deviations are more restrictive, and some are less restrictive, than the basic stress limits. Rules governing application of these special stress limits for Level C and Level D Service Limit applications are...
(b) Shearing Stress Only

(1) Bearing-Type Joints

(-a) Threads Excluded From Shear Planes. The allowable shear stress \( F_{vb} \) in bolts and threaded parts loaded in direct shear, expressed in ksi (MPa) of actual shear stress area available (applicable to the total nominal bolt area in the shear planes in this case), shall not exceed:

for ferritic steels:

\[
F_{vb} = \frac{0.625S_u}{3}
\]

for austenitic steels:

\[
F_{vb} = \frac{0.625S_u}{5}
\]

(-b) Threads Not Excluded From Shear Planes. The allowable shear stress \( F_{vb} \) in bolts and threaded parts loaded in direct shear, expressed in ksi (MPa) of actual shear stress area available (applicable to the total nominal bolt area in the shear planes in this case), shall not exceed:

for ferritic steels:

\[
F_{vb} = \frac{0.625S_u}{3}
\]

for austenitic steels:

\[
F_{vb} = \frac{0.625S_u}{5}
\]

(c) Combined Tensile and Shear Stresses

(1) Bearing-Type Joints. Bolts subjected to combined shear and tension shall be so proportioned that either the shear stress \( f_s \) or the tensile stress \( f_t \), expressed in ksi (MPa) of actual cross-sectional area, shall not exceed the value derived from the ellipse equation below when the corresponding computed tensile or shearing stress is substituted.

\[
\frac{f_t^2}{F_{lb}^2} + \frac{f_s^2}{F_{vb}^2} = 1
\]

The allowable tensile stress and shear stress values shall be those derived from the equations given in (a) and (b).

(2) Friction-Type Joints. A bolt in a connection designed as a friction-type joint is not subjected to shear (provided the joint does not slip into bearing); it experiences tension only. Friction-type joints shall be designed as given in (d).

(d) Slip Resistance — Friction-Type Joints. The maximum slip resistance to which a friction-type joint may be designed shall not exceed the value of \( P_s \), calculated in the following equation [see Table WD-3234.1(d)-1]:

\[
P_s = mnT_i k_s
\]

where

- \( k_s \) = the effective slip coefficient for the particular surface conditions taken from Table WD-3234.1(d)-1.
- \( m \) = the number of shear planes per bolt
- \( n \) = the number of bolts in the joint
- \( T_i \) = initial clamping force per bolt, lb (N)

<table>
<thead>
<tr>
<th>Surface Condition</th>
<th>Eff. Slip Coefficient, ( k_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean mill scale, grit-blasted heat treated steel and machined surfaces</td>
<td>0.25</td>
</tr>
<tr>
<td>Grit-blasted carbon and low alloy high-strength steel</td>
<td>0.41</td>
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<tr>
<td>Aluminum</td>
<td>0.45</td>
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Division 5 Update to Figure HBB-3221-1
Figure HBB-3221-1
Flow Diagram for Elevated Temperature Analysis

Load-Controlled Stress Limits

Design Limits
- Use design loads
- \( S_D \) → \( P_m \) → \( 1.5 S_D \) → \( P_L + P_B \)

Levels A and B Service Limits
- Use service loads
- \( S_{mt} \) or \( 1.2 S_m \) → \( P_m \) → \( P_L + P_B \) → \( P_L + P_B/K_t \)
- \( \sum \left( \frac{f_i}{f_{im}} \right) \leq \beta_i \)
- \( \sum \left( \frac{f_i}{f_{im}} \right) \leq 1.0 \)

Level C Service Limits
- \( S_L \) or \( 1.2 S_m \) → \( P_m \) → \( P_L + P_B \) → \( P_L + P_B/K_t \)
- \( \sum \left( \frac{f_i}{f_{im}} \right) \leq \beta_i \)
- \( \sum \left( \frac{f_i}{f_{im}} \right) \leq 1.0 \)

Level D Service Limits
- \( 0.67 S_L \) or \( 0.85 S_{fr} \) → \( P_m \) → \( P_L + P_B \) → \( P_L + P_B/K_t \)
- \( \sum \left( \frac{f_i}{f_{im}} \right) \leq \beta_i \)
- \( \sum \left( \frac{f_i}{f_{im}} \right) \leq 1.0 \)

Strain and Deformation Limits

- Check on time independent buckling
- (Division 1, NB-3333 or HBB-T-1500)

Functional Requirements

- Deformations
- Membrane strain limits
- 1% bending local
- Creep - fatigue evaluation
- \( \epsilon_{L_i} \)
- Elas. analy. test
- Material strain limit
- Assume \( \epsilon = 1\% \)
- Calculate inelastic strain
- \( n N_d + t T_d \)
- Buckling & instability
- \( \sum \left( \frac{f_i}{f_{im}} \right) \leq 1.0 \)

Legend
- Controlled quantity for elastic analysis
- Controlled quantity for inelastic analysis
- Computed quantity

No limits unless specified in the Design Specification

In Nonmandatory Appendix HBB-T
<table>
<thead>
<tr>
<th>Load-Controlled Stress Limits</th>
<th>Strain and Deformation Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Limits</td>
<td>Check on time independent buckling</td>
</tr>
<tr>
<td>Use design loads</td>
<td>(Division 1, NB-3133 or HBB-T-1500)</td>
</tr>
<tr>
<td>$P_m$</td>
<td><strong>Functional Requirements</strong></td>
</tr>
<tr>
<td>$S_o$</td>
<td>Deformations</td>
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<tr>
<td>$1.5S_o$</td>
<td>Limits in Design Specification</td>
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<td>$P_L + P_b$</td>
<td>Assume $\epsilon = 1%$</td>
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<tr>
<td>$P_m$</td>
<td>Calculate inelastic strain</td>
</tr>
<tr>
<td>$P + P_b$</td>
<td>Membrane Bending Local</td>
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<td>Creep - fatigue evaluation</td>
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<td>$\Sigma(\tau_i/\tau_i) \leq 1.00$</td>
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<tr>
<td>Legend</td>
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<td>Controlled quantity for inelastic analysis</td>
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<tr>
<td>Computed quantity</td>
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