(d) Calculate values of constants \(a\), \(b\), \(\beta\), and \(\phi\).

\[
\begin{align*}
    a &= 0.5D - r \\
    b &= L - r \\
    \beta &= \arccos\left(\frac{a}{b}\right), \text{radians} \\
    \phi &= \left(\sqrt{L^2 - r^2}\right) r, \text{radians}
\end{align*}
\]

(e) Calculate the value of \(c\).

If \(\phi\) is less than \(\beta\), then

\[
    c = a\left[\cos(\beta - \phi)\right]
\]

If \(\phi\) is equal to or greater than \(\beta\), then

\[
    c = a
\]

Determine the value of \(Re\).

\[
    Re = c + r
\]

(f) Calculate the value of internal pressure expected to produce elastic buckling, \(P_e\).

\[
P_e = \frac{S_d t_s}{C_2 R_e\left[\left(0.5R_e / r\right) - 1\right]}
\]

(g) Calculate the value of internal pressure expected to result in yield stress at the point of maximum stress, \(P_y\).

\[
P_y = \frac{S_d t_s}{C_2 R_e\left[\left(0.5R_e / r\right) - 1\right]}
\]

(h) Calculate the value of internal pressure expected to result in knuckle failure, \(P_{ck}\).

\[
P_{ck} = 0.6P_e \text{ for } P_e / P_y \leq 1.0 \\
P_{ck} = 0.408P_y + 0.192P_e \text{ for } 1.0 < P_e / P_y \leq 8.29 \\
P_{ck} = 2.0P_y \text{ for } P_e / P_y > 8.29
\]

(i) Calculate the value \(P_{ck}/1.5\). If \(P_{ck}/1.5\) is equal to or greater than the required internal design pressure \(P\), then the design is complete. If \(P_{ck}/1.5\) is less than the required internal design pressure \(P\), then increase the thickness and repeat the calculations.

(2) Design of Ellipsoidal Heads With \(t_s/L < 0.002\). The minimum required thickness of an ellipsoidal head having \(0.0005 \leq t_s/L < 0.002\) shall be larger of the thicknesses calculated by the equations in UG-32(c), or in (c) or (1). In using the equations in (1), the value of \(L\) is to be obtained from Table UG-37 and the value of \(r\) is to be obtained from Table 1-4.4.

### Table 1-4.2

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Table 1-4.2

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**Table 1-4.3**

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<tr>
<th>Material Is Listed</th>
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<tbody>
<tr>
<td>Table UCS-23</td>
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<tr>
<td>Table UNF-23.1</td>
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</tr>
<tr>
<td>Table UNF-23.2</td>
<td>150</td>
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<td>Table UNF-23.3</td>
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<tr>
<td>Table UNF-23.4</td>
<td>600</td>
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<td>Table UNF-23.5</td>
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<td>Table UHA-23</td>
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<tr>
<td>Table UHT-23</td>
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</tbody>
</table>

1-5 RULES FOR CONICAL REDUCER SECTIONS AND CONICAL HEADS UNDER INTERNAL PRESSURE

(a) The equations of (d) and (e) below provide for the design of reinforcement, if needed, at the large and small ends for conical reducer sections and conical heads where
all the elements have a common axis and the half-apex angle $\alpha \leq 30$ deg. Subparagraph (g) below provides for special analysis in the design of cone-to-cylinder intersections with or without reinforcing rings where $\alpha$ is greater than 30 deg.

In the design of reinforcement at the large and small ends of cones and conical reducers, the requirements of UG-41 shall be met.

(b) **Nomenclature**

$A_{eL}$ = effective area of reinforcement at large end intersection

$A_{eS}$ = effective area of reinforcement at small end intersection

$A_{rL}$ = required area of reinforcement at large end of cone

$A_{rS}$ = required area of reinforcement at small end of cone

$E_1$ = efficiency of longitudinal joint in cylinder. For compression (such as at large end of cone), $E_1 = 1.0$ for butt welds.

$E_2$ = efficiency of longitudinal joint in cone. For compression, $E_2 = 1.0$ for butt welds.

$E_c$ = modulus of elasticity of cone material

$E_r$ = modulus of elasticity of reinforcing ring material

$f_1$ = axial load per unit circumference at large end due to wind, dead load, etc., excluding pressure

$f_2$ = axial load per unit circumference at small end due to wind, dead load, etc., excluding pressure

$P$ = internal design pressure (see UG-21)

$Q_L$ = algebraical sum of $PR_L/2$ and $f_1$

$Q_S$ = algebraical sum of $PR_S/2$ and $f_2$

$R_L$ = inside radius of large cylinder at large end of cone

$R_S$ = inside radius of small cylinder at small end of cone

$S_r$ = allowable stress of reinforcing ring material at design temperature

$S_s$ = allowable stress of cylinder material at design temperature

$t$ = minimum required thickness of cylinder at cone-to-cylinder junction

$t_c$ = nominal thickness of cone at cone-to-cylinder junction

$t_r$ = minimum required thickness of cone at cone-to-cylinder junction

$y$ = cone-to-cylinder factor

= $S_r E_c$ for reinforcing ring on shell

= $S_c E_c$ for reinforcing ring on cone

$\Delta$ = angle indicating need for reinforcement at cone-to-cylinder junction having a half-apex angle $\alpha \leq 30$ deg. When $\Delta \geq \alpha$, no reinforcement is required at the junction (see Tables 1-5.1 and 1-5.2), deg.

$\alpha$ = half-apex angle of cone or conical section, deg.

(c) For conical reducers and conical heads, the following values shall be determined at large end and again at the small end in order that both the large end and the small end can be examined:

(1) When a cylinder having a minimum length of 2.0 $\sqrt{R_L E_1}$ is attached to the large end of the cone, determine $P/S_s E_1$ and then determine $\Delta$ at the large end from Table 1-5.1.

**NOTE:** If a cylinder is not present or does not meet the minimum length requirement, $\Delta$ is not calculated.

(2) When a cylinder having a minimum length of 1.4 $\sqrt{R_S E_1}$ is attached to the small end of the cone, determine $P/S_s E_1$ and then determine $\Delta$ at the small end from Table 1-5.2.

(3) Determine $k$:

$k = 1$ when additional area of reinforcement is not required

= $y/S_r E_r$ when a stiffening ring is required, but $k$ is not less than 1.0

(d) Reinforcement shall be provided at the large end of the cone when required by the following:

<table>
<thead>
<tr>
<th>$P/S_s E_1$</th>
<th>0.001</th>
<th>0.002</th>
<th>0.003</th>
<th>0.004</th>
<th>0.005</th>
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</thead>
<tbody>
<tr>
<td>$\Delta$, deg</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$P/S_s E_1$</th>
<th>0.006</th>
<th>0.007</th>
<th>0.008</th>
<th>0.009 [Note (1)]</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$, deg</td>
<td>25</td>
<td>27</td>
<td>28.5</td>
<td>30</td>
<td>...</td>
</tr>
</tbody>
</table>

**NOTE:**

(1) $\Delta = 30$ deg for greater values of $P/S_s E_1$. 

---

**Table 1-4.4**

<table>
<thead>
<tr>
<th>$D/2h$</th>
<th>$r/D$</th>
</tr>
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<tr>
<td>2.6</td>
<td>0.12</td>
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<tr>
<td>2.4</td>
<td>0.13</td>
</tr>
<tr>
<td>2.2</td>
<td>0.15</td>
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<td>2.0</td>
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<tr>
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<td>0.37</td>
</tr>
<tr>
<td>1.0</td>
<td>0.50</td>
</tr>
</tbody>
</table>

GENERAL NOTE: Interpolation permitted for intermediate values.

---

**Table 1-5.1**

<table>
<thead>
<tr>
<th>$P/S_s E_1$</th>
<th>0.001</th>
<th>0.002</th>
<th>0.003</th>
<th>0.004</th>
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<td>$\Delta$, deg</td>
<td>11</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P/S_s E_1$</th>
<th>0.006</th>
<th>0.007</th>
<th>0.008</th>
<th>0.009 [Note (1)]</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$, deg</td>
<td>25</td>
<td>27</td>
<td>28.5</td>
<td>30</td>
<td>...</td>
</tr>
</tbody>
</table>
For cones attached to a cylinder having a minimum length of 2.0 \( P_{/SE_1} \), reinforcement shall be provided at the junction of the cone with the large cylinder for conical heads and reducers without knuckles when the value of \( \Delta \) obtained from Table 1-5.1, using the appropriate ratio \( P_{/SE_1} \), is less than \( \alpha \). Interpolation may be made in the Table.

The required area of reinforcement shall be at least equal to that indicated by the following formula when \( Q_L \) is in tension:

\[
A_{RL} = \frac{kQ_L}{S_xE_1} \left(1 - \frac{\Delta}{\alpha}\right) \tan \alpha
\]

At the small end of the cone-to-cylinder juncture, the \( PR_{/2} \) term is in tension. When \( f_2 \) is in compression and the quantity is larger than the \( PR_{/2} \) term, the design shall be in accordance with U-2(g). The calculated localized stresses at the discontinuity shall not exceed the stress values specified in (g)(1) and (g)(2).

The effective area of reinforcement can be determined in accordance with the following formula:

\[
A_{el} = \left(t_c-t_f\right)\sqrt{R_{1c}\cos \alpha}
\]

Any additional area of reinforcement which is required shall be situated within a distance of \( \sqrt{R_{1c}} \) from the junction, and the centroid of the added area shall be within a distance of 0.25 \( \sqrt{R_{1c}} \) from the junction.

(2) For cones attached to flat covers, flanges, or other components where the length of cylinder, if present, is less than 1.4 \( \sqrt{R_{1c}} \), the required reinforcement shall be at least equal to that indicated by the following formula:

\[
A_{RS} = \frac{kQ_L R_s}{S_xE_2} \left(1 - \frac{\Delta}{\alpha}\right) \tan \alpha
\]

The effective area of reinforcement can be determined in accordance with the following formula:

\[
A_{es} = 0.78 \sqrt{R_{1s}} \left( t_s-t_f \right) / \cos \alpha
\]

Any additional area of reinforcement which is required shall be added to the cone.

(e) Reinforcement shall be provided at the small end of the cone when required by the following:

(1) For cones attached to a cylinder having a minimum length of 1.4 \( \sqrt{R_{1s}} \), reinforcement shall be provided at the junction of the conical shell of a reducer without a flare and the small cylinder when the value of \( \Delta \) obtained from Table 1-5.2, using the appropriate ratio \( P_{/SE_1} \), is less than \( \alpha \). Interpolation may be made in the Table.

The required area of reinforcement shall be at least equal to that indicated by the following formula when \( Q_s \) is in tension:

\[
A_{Rs} = \frac{kQ_L}{S_xE_1} \left(1 - \frac{\Delta}{\alpha}\right) \tan \alpha
\]

At the small end of the cone-to-cylinder juncture, the \( PR_{/2} \) term is in tension. When \( f_2 \) is in compression and the quantity is larger than the \( PR_{/2} \) term, the design shall be in accordance with U-2(g). The calculated localized stresses at the discontinuity shall not exceed the stress values specified in (g)(1) and (g)(2).

The effective area of reinforcement can be determined in accordance with the following formula:

\[
A_{es} = 0.78 \sqrt{R_{1s}} \left( t_s-t_f \right) / \cos \alpha
\]

Any additional area of reinforcement which is required shall be situated within a distance of \( \sqrt{R_{1s}} \) from the junction, and the centroid of the added area shall be within a distance of 0.25 \( \sqrt{R_{1s}} \) from the junction.

(1) For cones attached to a cylinder having a minimum length of 2.0 \( \sqrt{R_{1c}} \), reinforcement shall be provided at the junction of the cone with the large cylinder for conical heads and reducers without knuckles when the value of \( \Delta \) obtained from Table 1-5.1, using the appropriate ratio \( P_{/SE_1} \), is less than \( \alpha \). Interpolation may be made in the Table.

The required area of reinforcement shall be at least equal to that indicated by the following formula when \( Q_L \) is in tension:

\[
A_{RL} = \left(t_c-t_f\right)\sqrt{R_{1c}\cos \alpha}
\]

Any additional area of reinforcement which is required shall be added to the cone.

(f) Reducers not described in UG-36(e)(5), such as those made up of two or more conical frustums having different slopes, may be designed in accordance with (g).
(g) When the half-apex angle $\alpha$ is greater than 30 deg (0.52 rad), cone-to-cylinder junctions without a knuckle may be used, with or without reinforcing rings, if the design is based on special analysis, such as the beam-on-elastic-foundation analysis of Timoshenko, Hetenyi, or Watts and Lang. See U-2(g). When such an analysis is made, the calculated localized stresses at the discontinuity shall not exceed the following values:

(1) Membrane hoop stress plus average discontinuity hoop stress shall not be greater than 1.5 times the longitudinal stress at the surfaces.

(2) Membrane longitudinal stress plus discontinuity longitudinal stress due to bending shall not be greater than $S_{PS}$ [see UG-23(e)].

The angle joint (see 3-2) between the cone and cylinder shall be designed equivalent to a double butt-welded joint, and because of the high bending stress, there shall be no weak zones around the angle joint. The thickness of the cylinder may have to be increased to limit the difference in thickness so that the angle joint has a smooth contour.

### 1-6 DISHED COVERS (BOLTED HEADS)

(a) Dished heads with bolting flanges, both concave and convex to the pressure, and conforming to the several types illustrated in Figure 1-6, shall be designed in accordance with the equations which follow.

(b) The symbols used in the equations of this paragraph are defined as follows:

- $A =$ outside diameter of flange
- $B =$ inside diameter of flange
- $C =$ bolt circle, diameter
- $L =$ inside spherical or crown radius
- $M_o =$ the total moment determined as in 2-6 for heads concave to pressure and 2-11 for heads convex to pressure; except that for heads of the type shown in Figure 1-6, sketch (d), $H_D$ and $h_D$ shall be as defined below, and an additional moment $H_h$ (which may add or subtract) shall be included where $M_o$

$$H_D = \text{axial component of the membrane load in the spherical segment acting at the inside of the flange ring}$$

$$h_D = \text{radial distance from the bolt circle to the inside of the flange ring}$$

$$H_r = \text{radial component of the membrane load in the spherical segment acting at the intersection of the inside of the flange ring with the centerline of the dished cover thickness}$$

$$h_r = \text{lever arm of force } H_r \text{ about centroid of flange ring}$$

$$\beta_1 = \text{angle formed by the tangent to the centerline of the dished cover thickness at its point of intersection with the flange ring, and a line perpendicular to the axis of the dished cover}$$

$$= \arcsin \left( \frac{B}{2L + t} \right)$$

NOTE: Since $H_I$, in some cases will subtract from the total moment, the moment in the flange ring when the internal pressure is zero may be the determining loading for flange design.

$$P = \text{internal pressure (see UG-21) for the pressure on concave side, and external pressure for the pressure on convex side [see UG-28(f)]}$$

$$r = \text{inside knuckle radius}$$

$$S = \text{maximum allowable stress value (see UG-23)}$$

$$T = \text{flange thickness}$$

$$t = \text{minimum required thickness of head plate after forming}$$

(c) It is important to note that the actual value of the total moment $M_o$, may calculate to be either plus or minus for both the heads concave to pressure and the heads convex to pressure. However, for use in all of the equations that follow, the absolute values for both $P$ and $M_o$ are used.

(d) Heads of the type shown in Figure 1-6, sketch (a):

(1) the thickness of the head $t$ shall be determined by the appropriate formula in UG-32 for pressure on concave side, and UG-33 for pressure on convex side; the thickness of the skirt shall be determined by the formula for cylindrical shell in UG-27 for pressure on concave side and UG-28 for pressure on convex side;

(2) the head radius $L$ or the knuckle radius $r$ shall comply with the limitations given in UG-32;

(3) the flange shall comply at least with the requirements of Figure 2-4 and shall be designed in accordance with the provisions of 2-1 through 2-8 for pressure on concave side, and 2-11 for pressure on convex side. When a slip-on flange conforming to the standards listed in Table U-3 is used, design calculations per Mandatory Appendix 2 need not be done provided the design pressure-temperature is within the pressure-temperature rating permitted in the flange standard.

(e) Heads of the type shown in Figure 1-6, sketch (b) (no joint efficiency factor is required):

(1) head thickness

(-a) for pressure on concave side,

$$t = \frac{5PL}{6S} \quad (1)$$

(-b) for pressure on convex side, the head thickness shall be determined based on UG-33(c) using the outside radius of the spherical head segment;
1-8 RULES FOR REINFORCEMENT OF CONES AND CONICAL REDUCERS UNDER EXTERNAL PRESSURE

(a) The equations of (b) and (c) below provide for the design of reinforcement, if needed, at the cone-to-cylinder junctures for reducer sections and conical heads where all the elements have a common axis and the half-apex angle \( \alpha \leq 60 \) deg. Subparagraph (e) below provides for special analysis in the design of cone-to-cylinder intersections with or without reinforcing rings where \( \alpha \) is greater than 60 deg.

In the design of reinforcement for a cone-to-cylinder juncture, the requirements of UG-41 shall be met.

The nomenclature given below is used in the equations of the following subparagraphs:

\[
\begin{align*}
A & = \text{factor determined from Section II, Part D, Subpart 3, Figure G and used to enter the applicable material chart in Section II, Part D, Subpart 3} \\
A_{eL} & = \text{effective area of reinforcement at large end intersection} \\
A_{eS} & = \text{effective area of reinforcement at small end intersection} \\
A_{rL} & = \text{required area of reinforcement at large end of cone} \\
A_{rS} & = \text{required area of reinforcement at small end of cone} \\
A_e & = \text{cross-sectional area of the stiffening ring} \\
A_T & = \text{equivalent area of cylinder, cone, and stiffening ring, where} \\
A_{TL} & = \frac{L_f L_S}{2} + \frac{L_c L_c}{2} + A_e \text{for large end} \\
A_{TS} & = \frac{L_f L_S}{2} + \frac{L_c L_c}{2} + A_e \text{for small end} \\
B & = \text{factor determined from the applicable material chart in Section II, Part D, Subpart 3 for maximum design metal temperature [see UG-20(c)]} \\
D_L & = \text{outside diameter of large end of conical section under consideration} \\
D_o & = \text{outside diameter of cylindrical shell (In conical shell calculations, the value of } D_o \text{ and } D_L \text{ should be used in calculations in place of } D_o \text{ depending on whether the small end } D_o \text{ or large end } D_L \text{ is being examined.)} \\
D_s & = \text{outside diameter at small end of conical section under consideration} \\
E_1 & = \text{efficiency of longitudinal joint in cylinder. For compression (such as at small end of cone), } E_1 = 1.0 \text{ for butt welds.} \\
E_2 & = \text{efficiency of longitudinal joint in cone. For compression, } E_2 = 1.0 \text{ for butt welds.} \\
E_c & = \text{modulus of elasticity of cone material} \\
E_r & = \text{modulus of elasticity of stiffening ring material} \\
E_s & = \text{modulus of elasticity of shell material} \\
E_x & = E_c, E_r, \text{ or } E_s \\
f_1 & = \text{axial load per unit circumference at large end due to wind, dead load, etc., excluding pressure} \\
f_2 & = \text{axial load per unit circumference at small end due to wind, dead load, etc., excluding pressure} \\
l & = \text{available moment of inertia of the stiffening ring cross section about its neutral axis parallel to the axis of the shell} \\
l' & = \text{available moment of inertia of combined shell-cone or ring-shell-cone cross section about its neutral axis parallel to the axis of the shell. The nominal shell thickness, } t_n \text{, shall be used, and the width of the shell which is taken as contributing to the moment of inertia of the combined section shall not be greater than } 1.10 \sqrt{D_L t_n} \text{ and shall be taken as lying one-half on each side of the cone-to-cylinder junction or of the centroid of the ring. Portions of the shell plate shall not be considered as contributing area to more than one stiffening ring.} \\
\text{CAUTION: Stiffening rings may be subject to lateral buckling. This should be considered in addition to the requirements for } I_s \text{ and } l' \text{, [see U-2(g)].} \\
l_s & = \text{required moment of inertia of the stiffening ring cross section about its neutral axis parallel to the axis of the shell} \\
l'_s & = \text{required moment of inertia of the combined shell-cone or ring-shell-cone cross section about its neutral axis parallel to the axis of the shell} \\
k & = 1 \text{ when additional area of reinforcement is not required} \\
& = y/S.E_s \text{ when a stiffening ring is required, but } k \text{ is not less than } 1.0 \\
L & = \text{axial length of cone} \\
L_c & = \text{length of cone between stiffening rings measured along surface of cone, in. (mm). For cones without intermediate stiffeners,} \\
& = \sqrt{L^2 + (R_L - R_o)^2} \\
L_L & = \text{design length of a vessel section taken as the largest of the following:} \\
& = \text{(a) the center-to-center distance between the cone-to-large-shell junction and an adjacent stiffening ring on the large shell;} \\
& = \text{(b) the distance between the cone-to-large-shell junction and one-third the depth of head on the other end of the large shell if no other stiffening rings are used.} \\
l_s & = \text{design length of a vessel section taken as the largest of the following:} \\
& = \text{(a) the center-to-center distance between the cone-to-small-shell junction and adjacent stiffening ring on the small shell;} \\
& = \text{(b) the distance between the cone-to-small-shell junction and one-third the depth of head on the other end of the small shell if no other stiffening rings are used.}


\[ \Delta = 104 \sqrt{\frac{P}{S_x E_1}} \]

NOTE: If the stiffeners should be so located that the maximum permissible effective shell sections overlap on either or both sides of a stiffener, the effective shell section for that stiffener shall be shortened by one-half of each overlap.

\[ P = \text{external design pressure} \]
\[ Q_L = \text{algebraical sum of } PR_{1/2} \text{ and } f_1 \]
\[ Q_s = \text{algebraical sum of } PR_{1/2} \text{ and } f_2 \]
\[ R_L = \text{outside radius of large cylinder} \]
\[ R_s = \text{outside radius of small cylinder} \]
\[ S_c = \text{allowable stress of cone material at design temperature} \]
\[ S_{Es} = \text{allowable stress of stiffening ring material at design temperature} \]
\[ t = \text{minimum required thickness of cylinder at cone-cylinder junction} \]
\[ t_{sc} = \text{nominal thickness of cone at cone-cylinder junction} \]
\[ t_{ss} = \text{nominal thickness of cylinder at cone-cylinder junction} \]
\[ y = \text{cone-to-cylinder factor} \]
\[ = S_{Es} \text{ for stiffening ring on shell} \]
\[ = S_{Es} \text{ for stiffening ring on cone} \]
\[ \alpha = \text{one-half the included (apex) angle of the cone at the centerline of the head} \]
\[ \Delta = \text{value to indicate need for reinforcement at cone-cylinder intersection having a half-apex angle } \alpha \leq 60 \text{ deg. When } \Delta > \alpha, \text{ no reinforcement is required at the junction (see Table 1-8.1).} \]

(b) Reinforcement shall be provided at the large end of the cone when required by (1) or (2). When the large end of the cone lies within a distance of half the included (apex) angle of the cone at the centerline of the head from the junction, the required reinforcement shall be obtained from Table 1-8.1 using the appropriate ratio \( P/S_x E_1 \) is less than \( \alpha \). Interpolation may be made in the Table.

The required area of reinforcement shall be at least equal to that indicated by the following formula when \( Q_L \) is in compression:

\[ A_{RL} = \frac{kQ_L \tan \alpha}{S_{Es} E_1} \left[ 1 - \frac{1}{4} \left( \frac{PR_{1/2} - Q_s}{Q_s} \right) \Delta \right] \]

At the large end of the cone-to-cylinder juncture, the \( PR_{1/2} \) term is in compression. When \( f_1 \) is in tension and the quantity is larger than the \( PR_{1/2} \) term, the design shall be in accordance with U-2(g). The calculated localized stresses at the discontinuity shall not exceed the stress values specified in 1-5(g)(1) and 1-5(g)(2).

The effective area of reinforcement can be determined in accordance with the following formula:

\[ A_{RL} = 0.55 \sqrt{D_L t_{c}} (t_{c} - t_{c}/\cos \alpha) \]

Any additional area of stiffening which is required shall be situated within a distance of \( \sqrt{R_L t_s} \) from the junction of the reducer and the cylinder. The centroid of the added area shall be within a distance of \( 0.25 \times \sqrt{R_L t_s} \) from the junction.

(2) For cones attached to flat covers, flanges, or other components where the length of cylinder, if present, is less than \( 2.0 \times \sqrt{R_L t_s} \), the required reinforcement shall be at least equal to that indicated by the following formula:

\[ A_{RL} = \frac{kQ_L D_L}{2S_{Es} E_2} \tan \alpha \]

The effective area of reinforcement can be determined in accordance with the following formula:

\[ A_{RL} = 0.55 \sqrt{D_L t_{c}} (t_{c} - t_{c}/\cos \alpha) \]

Any additional area of reinforcement that is required shall be added to the cone.

(3) When the cone-to-cylinder or knuckle-to-cylinder juncture is a line of support, the moment of inertia for a stiffening ring at the large end shall be determined by the procedure in Steps 1 through 8 below.

For cones attached to flat covers, flanges, or other components where the length of cylinder, if present, is less than \( 2.0 \times \sqrt{R_L t_s} \), length \( L_L \) in the formulas for \( A_{TL} \) and \( M \) shall be zero.

Step 1. Assuming that the shell has been designed and \( D_L, L_L, \) and \( t \) are known, select a member to be used for the stiffening ring and determine cross-sectional area

<table>
<thead>
<tr>
<th>( P/S_x E_1 )</th>
<th>[ \Delta ]</th>
<th>[ \Delta ]</th>
<th>[ \Delta ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>0.0005</td>
<td>0.010</td>
<td>0.02</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>0.04</td>
<td>0.08</td>
<td>0.10</td>
<td>0.125</td>
</tr>
<tr>
<td>21</td>
<td>29</td>
<td>33</td>
<td>37</td>
</tr>
<tr>
<td>0.2</td>
<td>0.25</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>47</td>
<td>52</td>
<td>57</td>
<td>60</td>
</tr>
</tbody>
</table>

NOTE: \( \Delta = 60 \text{ deg for greater values of } P/S_x E_1 \)
Then calculate factor $B$ using the following formula. If $F_L$ is a negative number, the design shall be in accordance with U-2(g):

$$B = 3\left(\frac{F LD_t}{A_{TL}}\right)$$

where

$$F_L = PM + f_1 \tan \alpha$$

$$M = \frac{-R_2 \tan \alpha}{2} + \frac{l_4}{2} + \frac{R_2^2 - R_t^2}{3R_2 \tan \alpha}$$

Step 2. Enter the right-hand side of the applicable material chart in Section II, Part D, Subpart 3 for the material under consideration at the value of $B$ determined by Step 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of $A$ in Step 4 below.

Step 3. Move horizontally to the left to the material/temperature line for the design metal temperature. For values of $B$ falling below the left end of the material/temperature line, see Step 5 below.

Step 4. Move vertically to the bottom of the chart and read the value of $A$.

Step 5. For value of $B$ falling below the left end of the material/temperature line for the design temperature, the value of $A$ can be calculated using the formula $A = 2B/E_x$ for $B$ above the material/temperature line for the design temperature. For values of $B$ having multiple values of $A$, such as when $B$ falls on a horizontal portion of the curve, the smallest value of $A$ shall be used.

Step 6. Compute the value of the required moment of inertia from the equations for $I_c$ or $I'_c$. For the circumferential stiffening ring only,

$$I_c = \frac{AD_t^2 A_{TL}}{14.0}$$

For the shell

$$A_{ec} = 0.55 \sqrt{\frac{D_t t_c}{\cos \alpha}} (t_c - t_r)$$

$$I'_c = \frac{AD_t^2 A_{TL}}{10.9}$$

Step 7. Determine the available moment of inertia of the ring only $I'_c$ or the shell-cone or ring-shell-cone $I'_c$.

Step 8. When the ring only is used,

$$I \geq I_c$$

and when the shell-cone or ring-shell-cone is used,

$$I \geq I'_c$$

If the equation is not satisfied, a new section with a larger moment of inertia must be selected, and the calculation shall be done again until the equation is met.

The requirements of UG-29(b), UG-29(c), UG-29(d), UG-29(e), and UG-29(f) and UG-30 are to be met in attaching stiffening rings to the shell.

(c) Reinforcement shall be provided at the small end of the cone when required by (1) or (2). When the small end of the cone is considered a line of support, the moment of inertia for a stiffening ring shall be determined in accordance with (3).

(1) For cones attached to a cylinder having a minimum length of $1.4\sqrt{R_s E_x}$, reinforcement shall be provided at the junction of the conical shell of a reducer without a flange and the small cylinder. The required area of reinforcement shall be at least equal to that indicated by the following formula when $Q_s$ is in compression:

$$A_{fs} = \frac{kQ_s R_t \tan \alpha}{S_3 \mu_1}$$

At the small end of the cone-to-cylinder juncture, the $PR_s/2$ term is in compression. When $f_2$ is in tension and the quantity is larger than the $PR_s/2$ term, the design shall be in accordance with U-2(g). The calculated localized stresses at the discontinuity shall not exceed the stress values specified in 1-5(g)(1) and 1-5(g)(2).

The effective area of reinforcement can be determined in accordance with the following formula:

$$A_{es} = 0.55 \sqrt{D_t t_c} (t_c - t_r) / \cos \alpha$$

Any additional area of stiffener which is required shall be situated within a distance of $0.25\sqrt{R_s E_x}$ from the junction.

(2) For cones attached to flat covers, flanges, or other components where the length of cylinder, if present, is less than $1.4\sqrt{R_s E_x}$, the required reinforcement shall be at least equal to that indicated by the following formula:

$$A_{es} = \frac{kQ_s D_t}{2S_3 E_x}$$

The effective area of reinforcement can be determined in accordance with the following formula:

$$A_{es} = 0.55 \sqrt{D_t t_c} (t_c - t_r) / \cos \alpha$$

Any additional area of reinforcement that is required shall be added to the cone or the flange.
(3) When the cone-to-cylinder or knuckle-to-cylinder juncture is a line of support, the moment of inertia for a stiffening ring at the small end shall be determined by the procedure in Steps 1 through 8 below.

For cones attached to flat covers, flanges, or other components where the length of the cylinder, if present, is less than 1.4 \( \sqrt{R_t L_s} \), length \( L_s \) in the formulas for \( A_{TS} \) and \( N \) shall be zero.

**Step 1.** Assuming that the shell has been designed and \( D_s, L_s, \) and \( t \) are known, select a member to be used for the stiffening ring and determine cross-sectional area \( A_{TS} \). Then calculate factor \( B \) using the following formula. If \( F_s \) is a negative number, the design shall be in accordance with U-2(g):

\[
B = \frac{3}{4} \left( \frac{F_s D_s}{A_{TS}} \right)
\]

where

\[
F_s = PN + f_s \tan \alpha
\]

\[
N = \frac{R_t \tan \alpha}{2} + \frac{L_s}{2} + \frac{R_t^2}{6} R_s \tan \alpha
\]

**Step 2.** Enter the right-hand side of the applicable material chart in Section II, Part D, Subpart 3 for the material under consideration at the value of 0.2 determined by Step 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of \( A \) in Step 4 below.

**Step 3.** Move horizontally to the left to the material/temperature line for the design metal temperature. For values of \( B \) falling below the left end of the material/temperature line, see Step 5 below.

**Step 4.** Move vertically to the bottom of the chart and read the value of \( A \).

**Step 5.** For values of \( B \) falling below the left end of the material/temperature line for the design temperature, the value of \( A \) can be calculated using the formula \( A = 2B/E_s \). For value of \( B \) above the material/temperature line for the design temperature, the design shall be either per U-2(g) or by changing the cone or cylinder configuration, stiffening ring location on the shell, and/or reducing the axial compressive force to reduce the \( B \) value to below or at the material/temperature line for the design temperature. For values of \( B \) having multiple values of \( A \), such as when \( B \) falls on a horizontal portion of the curve, the smallest value of \( A \) shall be used.

**Step 6.** Compute the value of the required moment of inertia from the equations for \( I_s \) or \( I'_{s} \).

For the circumferential stiffening ring only,

\[
I_s = \frac{AD_s^2 A_{TS}}{14.0}
\]

For the shell-cone or ring-shell-cone section,

\[
I'_s = \frac{AD_s^2 A_{TS}}{10.9}
\]

**Step 7.** Determine the available moment of inertia of the ring only \( I \) or the shell-cone or ring-shell-cone \( I' \).

**Step 8.** When the ring only is used,

\[
I \geq I_s
\]

and when the shell-cone or ring-shell-cone is used:

\[
I' \geq I'_s
\]

If the equation is not satisfied, a new section with a larger moment of inertia must be selected, and the calculation shall be done again until the equation is met.

The requirements of UG-29(b), UG-29(c), UG-29(d), UG-29(e), and UG-29(f) and UG-30 are to be met in attaching stiffening rings to the shell.

(d) Reducers not described in UG-36(e)(5), such as those made up of two or more conical frustums having different slopes, may be designed in accordance with (e).

(e) When the half-apex angle \( \alpha \) is greater than 60 deg (1.1 rad), cone-to-cylinder junctions without a knuckle may be used, with or without reinforcing rings, if the design is based on special analysis, such as the beam-on-elastic-foundation analysis of Timoshenko, Hetenyi, or Watts and Lang. See U-2(g). The effect of shell and cone buckling on the required area and moment of inertia at the joint is to be taken into consideration in the analysis. When such an analysis is made, the calculated localized stresses at the discontinuity shall not exceed the following values:

(1) Membrane hoop stress plus average discontinuity hoop stress shall not be greater than 1.55.

(2) Membrane longitudinal stress plus discontinuity longitudinal stress due to bending shall not be greater than \( S_{PS} \) [see UG-23(e)], where the “average discontinuity hoop stress” is the average hoop stress across the wall thickness due to the discontinuity at the junction, disregarding the effect of Poisson’s ratio times the longitudinal stress at the surfaces.