which are simply supported or clamped at both ends. Of course, in determining the RMS response with eq. N-1342.1(94), the mode shapes corresponding to the actual boundary conditions and normalized according to eq. N-1342.1(86) are used. For other boundary conditions and higher modes, the joint acceptance integral will have to be evaluated either numerically (most cases) or in closed form from eq. N-1342.1(91). Since $j_{ij}^2 \leq 1.0$, (ref. [127]), an upper bound response estimate can be found by setting all the $j_{ij}^2 = 1.0$ in eq. N-1342.1(94).

The random characteristics of the forces exerted on the tubes by the turbulent flow must be obtained from tests. Two expressions for the power spectral density of the turbulent force per unit length on tubes in a tube array are:

$$G(f) = \left[ C(f) \rho v^2_d D / \pi^2 \right]^2$$  \hspace{1cm} (96)

(ref. [100]) and

$$G(f) = \left[ C(f) \rho v^2_d D / \pi^2 \right]^2 \left( D / v_g \right)$$  \hspace{1cm} (97)

(ref. [128]) where the gap velocity $v_g$ is related to the velocity upstream of the tubes, $v_o$, by

$$v_g / v_o = \rho / (\rho - D)$$  \hspace{1cm} (98)

The coefficient $C(f)$ in eq. (96) has units of $sec^{-3/2}$ and is given in Figure N-1343-1 as a function of the frequency $f$. Therefore, the application of eq. (96) should be limited to the parameter range for which the data were taken, namely, high turbulent water flow (1 to 2 m/sec) entering closely spaced heat exchanger tubes of 12 to 19 mm in diameter. The decrease in $C(f)$ with penetration into the bundle is attributed to the highly turbulent inlet flow and possible vortex excitation observed in the first few tube rows. Data for the nondimensional lift coefficient $C_L(f)$ of eq. (97) have not been obtained for as many tube array configurations (refs. [141] and [142]), but use of this alternative expression may predict less conservative responses (ref. [142]).

For an isolated tube in cross flow, which is not subject to conditions of lock-in vortex shedding (see N-1332), the power spectral density $G(f)$ and the correlation length are strong functions of the turbulence created in the incident flow stream by the upstream structures. Relatively small amounts of turbulence can cause significant reductions in the effectiveness of vortex shedding as an excitation mechanism, and all periodicity can be eliminated with sufficiently strong incident turbulence. For given turbulence intensities and scale lengths of the incident flow, $G(f)$ is available (ref. [129]) and the correlation length $L_c$ may be approximated by the scale length of the incident flow. In the absence of specific information about the incident flow, the random turbulence coefficient for the upstream tube in Figure N-1343-1(a) can be used to estimate $G(f)$ for most isolated tubes in cross flow because of the wide variety of incident flow conditions contained in the data base. In the latter case, the velocity used in eqs (96) and (97) should be the free stream velocity of the flow.

If upstream structures produce well defined vortices, strong excitation mechanisms may be created on isolated cylinders more than twenty diameters downstream (ref. [130]). Such configurations should be avoided.

**N-1343.2 Multiple Spans of Uniform Cross Flow.** In many applications, a cylinder is subject to one or more partial spans of uniform, but different, velocity cross flows that are uncorrelated with each other over the remainder of the cylinder's length. Conditions often exist when baffles are used to direct different density flows in the interior of the pressure vessel (heat exchangers, reactors, etc.). The mean square response for such conditions can be determined by simple generalizations of the results given in N-1342.2 for homogeneous turbulence excitation.

Since the uniform cross flow over the span of length $L_i$ is uncorrelated with the uniform cross flows over the other spans, $G(f)$ in eq. N-1342.1(94) can be calculated (ref. [127]) by summing the products of the locally defined spectra $G(f)$ and joint acceptances $j_{ij}^2$ over all the spans $i$ over which there is significant cross flow. Thus, the mean square response becomes

$$\sigma^2_x \approx \sum_i \sum \frac{L_i G(f_i) \Phi_i^2(x)}{64 \pi^2 \frac{M_i^2}{E_i^2} \xi_i}$$  \hspace{1cm} (99)

where

$$G(f_i) = G(f_i) \int_0^T \Phi_i(x) dx$$  \hspace{1cm} (100)

(1) are determined with eq. N-1342.1(88) using that part of $\Phi_i$, denoted by $\Phi_i$, that is active over the $i$th span with length $L_i$. As discussed in N-1343.1, if $\Phi_i$ is similar to the fundamental mode shape of a one-span beam with simple or clamped supports at each end, then $\sigma^2_x \approx C_i / L_i$. The correlation lengths inside a tube bundle are smaller than that for an isolated tube, being about 1-2 tube diameters. After specifying $G(f)$ and using eqs. N-1343.1(96) and N-1343.1(97) for instance, the mean square response can be determined.

**N-1343.3 Nonuniform Cross Flow.** In industrial heat exchangers, the cross flow velocities are seldom uniform over the entire length, or even one span of the tubes. While an average cross flow velocity can be used to estimate the
ACOUSTIC MODES OF A FLUID ANNULUS BONDED BY RIGID WALLS

The natural frequencies associated with the \((\alpha, \beta)\)th mode of the fluctuation pressure distribution

\[
\rho_{\alpha \beta} = R_{\alpha \beta}(x) \phi(x) \cos \beta \theta
\]  

(106)

inside the fluid annulus are given by the roots of the equation (ref. [153]),

\[
J_\beta(x_{\alpha})Y_\beta(x_{\beta}) = J_\beta(x_{\beta})Y_\beta(x_{\alpha})
\]  

(107)

where \(J, Y\) are Bessel functions and,

\[
x_{\alpha}^2 = \left( \frac{w}{c} \right)^2 - \left( \frac{\kappa}{\ell} \right)^2 > 0
\]  

(108)

where

\[
\kappa = \alpha \text{ if the fluid annulus has both pressure released (p = 0) or both hard (p = max.) ends}
\]

\[
= (2\alpha - 1)/2 \text{ if one end is pressure released and the other end is hard.}
\]

Note that acoustic modes exist in the fluid annulus only if,

\[
\omega/c > \kappa \pi / \ell
\]

The frequency at which,

\[
\omega/c = \kappa \pi / \ell
\]

is known in acoustics as the coincidence frequency.

FREQUENCY EQUATION AND MODE SHAPE FOR A THIN FLUID ANNULUS

For many applications, the fluid annulus is thin compared with its radius, so that the condition \((b - a)/a > 5\) is satisfied. Under this condition, the following approximate equation for the natural frequencies holds true for the plane wave mode (no radial node).

\[
f_{\alpha \beta} = \left( c/2\pi \right)^{1/2} \left[ \left( \kappa \pi / \ell \right)^2 + \left( 2\beta / (a + b) \right)^2 \right]^{1/2}
\]

The second radial mode (with one radial node) usually has a lowest frequency much higher than that of the fundamental radial mode up to \(\alpha = \beta = 5\). Thus, it can be ignored if only the lowest twenty or so acoustic modes are included in the analysis.

FREE VIBRATION OF COUPLED FLUID-SHELL SYSTEMS

When the cylindrical shells bounding the fluid annulus are flexible, then not only are the motions of the shells coupled to the fluid, but they are coupled together by the fluid between them. It was shown that (ref. [145]) there is no cross circumferential model coupling between the fluid and a system of two coaxial, circular cylindrical shells. However, except in the rare case when both the cylinder and the fluid can be represented by the same mode shape function, as is the case of a simply supported cylinder vibrating in a fluid annulus with open ends, there will be coupling between the axial structural and acoustic modes (refs. [93] and [145]).

THE HYDRODYNAMIC MASS MATRIX

It was shown that for small motions of the shell, the pressure induced by the structure in the fluid is proportional to the normal component of the acceleration \(\ddot{w}\), of shell (refs. [93], [145], and [155]):

\[
\begin{pmatrix} p \\ \ddot{w} \end{pmatrix} = \begin{pmatrix} H \end{pmatrix} \begin{pmatrix} \dot{w} \end{pmatrix}
\]  

(109)

The "constant" of proportionality is commonly called the hydrodynamic mass or added mass matrix. In general, it is a full matrix, the element of which is dependent on the frequency. For two coaxial cylindrical shells coupled by a fluid gap, the hydrodynamic mass matrix elements are (refs. [93] and [145]).

\[
\begin{align*}
H_{mn} &= \sum_{\alpha} c_{\alpha m} c_{\alpha n} \psi_{\alpha}^{ab} \\
H_{mn} &= \sum_{\alpha} c_{\alpha m} c_{\alpha n} \psi_{\alpha}^{ba}
\end{align*}
\]

(110)

where \(c_{\alpha m}^{ab}\), for example, is the projection of the \(m\)th axial mode shape, \(\psi_{\alpha}\) of cylinder \(a\) (the inner cylinder) onto the \(\alpha\)th axial acoustic mode shape \(\phi_{\alpha}\) of the fluid annulus:

\[
c_{\alpha m}^{ab} = \int_{0}^{\ell} \psi_{\alpha}(x) \phi_{\alpha}(x) dx
\]  

(111)

\(c_{\alpha m}^{ab}\) are similarly defined. \(h\) (and hence \(H\)) are in units of mass per unit area.

The general expressions for the hydrodynamic mass which are valid for any boundary condition and any \(D/\ell\) ratio, are quite complicated, but still can easily be computed with the aid of a personal computer. In the following paragraphs, simplified expressions for computing \(h\) are given for several commonly encountered special cases.

SLENDER CYLINDER APPROXIMATION. When the conditions \(|s \eta a/\ell - \omega a/c| < 1\) and \(|s \eta b/\ell - \omega b/c| < 1\) are simultaneously satisfied, ref. [145] shows that:

\[
\begin{align*}
h_{an}^{a} &= \left( pa/\eta \right) \left( b^{2n} + a^{2n} \right) / \left( \ell^{2n} - a^{2n} \right) \\
h_{an}^{ab} &= - \left( 2 pb / \eta \right) \left( a^{2n} b^{2n} \right) / \left( \ell^{2n} - a^{2n} \right) \\
h_{an}^{b} &= \left( pb / \eta \right) \left( b^{2n} + a^{2n} \right) / \left( \ell^{2n} - a^{2n} \right) \\
h_{an}^{ba} &= - \left( 2 pb / \eta \right) \left( a^{2n} b^{2n} \right) / \left( \ell^{2n} - a^{2n} \right)
\end{align*}
\]  

(112)


